AUTOMATED ECONOMIC DESIGN OF LINEAR DISCRETE STRUCTURES

A THESIS

SUBMITTED TO THE FACULTY

OF

INDIAN INSTITUTE OF TECHNOLOGY, KANPUR

BY

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IN PARTIAL FULFILMENT OF THE

REQUIREMENTS FOR THE DEGREE

OF

MASTER OF TECHNOLOGY

JUNE 1968

DEDICATED TO MY PARENTS

ACKNOWLEDGEMENTS

The author wishes to express his indebtedness and gratitude to his advisor Dr. A.V. Setlur for his continuous suggestions in the development of this thesis. His encouragements, splendid advice and his patience were very much necessary for the completion of this thesis.

Also I gratefully acknowledge my sincere thanks and appreciation for the excellent suggestions and advice of Dr. P. Dayaratnam.

Thankfulness to the staff of Computer Centre, Indian Institute of Technology, Kanpur for their excellent cooperation.

Gratefulness to Shri Omesh Abhat, graduate student, for his suggestive criticism and recognization of the typing of the manuscript by Shri Rameshwar Lal.

CONTENTS

ABSTRACT

In this dissertation an automatic economic design of linear discrete structures is presented. Two types of such structures are considered: (a) pin-jointed plane and space trusses, and (b) rigid jointed plane frames which include continuous beams, gable bents, multi-storeyed buildings and arches. For both types of structures, the analysis is done by point iteration of the generalised slope deflection equations.

For both cases feasibility of structural design is examined. Pin-jointed trusses are designed for single as well as multi-load conditions by an iterative procedure such that the volume of the structural members is a minimum. The resulting structure need not be a fully stressed one.

Rigid jointed structural members are designed to achieve the minimum sectional area by iterating on the depth of the section. The sections used are prismatic I-section members.

The design procedure is completely automated once the geometry and design loads are specified.

The structure is first analysed by approximate method and approximate section properties are arrived at. The properties are, in turn, utilized in the more exact analysis and the design cycle is repeated until an economic depth and corresponding section property for every member is obtained.

Several problems are illustrated and discussions of the results are presented.

NOMENCLATURE*

A	Area of cross-section
A_f	Area of flange
b	Spread of flange
BETA	Interaction value
BM	Beam bending moment
BS	Beam shear force
CM	Column moment
CM_r	Column bending moment for rth storey
d	Depth of section
E	Young's modulus
F_a, F_b	Allowable stress in axial and bending
f_a, f_b	Actual stress due to axial force and bending moment
F_{ij}	Force in I, J member
h	Depth of a section
I	Moment of inertia and used as a subscript
I_r	Moment of inertia of the columns of rth storey
I_c	Column moment of inertia
I_g	Girder moment of inertia
I_k	Moment of inertia about k axis
J	Polar moment of inertia

*Also see nomenclature for computer programmes

K	Stiffness matrix
K_{ij}, K_{ji}	Element stiffness matrices
L	Span of member
L_c, L_g	Column and girder lengths
M	Bending moment
M_r	Moment of resistance of 4rth storey
N_r	Column normal force for rth storey
N	Actual force
P	Load at a joint
R_i	Distance from C-G. of a column of ith storey
R_{ij}	Reaction of ij joint
S_{cr}	Critical stress to prevent failure due to buckling
s_{cr}	Critical bending stress to prevent failure due to bending buckling
T	Transformation matrix
t_1, t_a, t_b	Thickness to prevent buckling due to shear axial and bending moments
U	Jont displacement
V	Shear force
V_r	Shear at rth storey
v_{cr}	Critical value of shear stress to prevent buckling
v_{al}	Allowable shear stress

V_{max}	Design shear force
w	Inspan load
Y_i	Distance of a section from neutral axis
Z	Section modulus
$\theta, \theta_r .., \theta_o$	Rotation of a joint
Δ, δ	Displacement of a joint
ν	Poissions Ratio
ϕ	Storey sway angle
σ	Stress

CHAPTER I

INTRODUCTION

The structural design may be defined as the process of determining the most suitable geometry of a structure to satisfy a set of purposes and to dimension the structural elements of which it is composed of. A most suitable geometry has to satisfy the functional, economic, social, aesthetic and many more requirements. Once the configuration is obtained, the forces or loads that the structure has to resist is to be determined. These loads are tabulated in the Codes of Specifications and does not require much attention for the conventional type of structures, but may really be a difficult problem for special and rare type of structures like space vehicles, supersonic jet planes etc. It is beyond the scope of this thesis either to design the geometry or to determine the design forces, instead both configuration and design load systems are assumed. So the problem dealt here is to proportion the elements of the structure subjected to a given set of loads. The structures considered are the rigid jointed plane frames, like multi-storeyed building, gable frames and beams; and pin jointed plane and space trusses. Plane arches may approximately be solved if the curved geometry of the arch is replaced by an equivalent linear geometry. The length of

the arch along the centre line may be divided into a set of linear elements for this purpose.

1.1 Design of Elements

There are two distinct phases in the design of the members of a structure:

Phase 1:

The stress and deformation analysis of the entire structure has to be completed and design forces for elements obtained. The stress analysis depends upon a set of equations. Henceforth these equations will be called as the equations of analysis. These are the equilibrium equations which state that the internal forces in the structure and the external loads must be in equilibrium and the equations of compatability which states that the deformed members of the structure must continue to fit together, so that the deformations are geometrically compatible. There is another set of equations called the stress strain laws. Since the structures dealt here are linear structures these equations are automatically satisfied and do not appear in the analysis.

Phase 2:

The members are to be designed for the above forces of the analysis such that the stress at the highly stressed sections should be within the allowable limits. The deformations at any point along the span of the member should also be within the permissible limits. This is the elastic design

of structures. For future references these will be known as the constraints of the design. Several more constraints on depth, width and thickness of the section may be laid down.

In short any valid design of elements must satisfy the equations of analysis and the constraints of design.

The above design does not ensure that the volume of the structure is a minimum. If in addition to the above equations and constraints one more constraint of the volume i.e. the total volume of the structure be a minimum is laid down the problem becomes a minimum weight design problem. Mathematical programming has to be resorted to for the solution of thisproblem. If all the equations of analysis and constraints of design are linear then it is a linear programming problem otherwise it is a non-linear one if the equations or constraints are not linear. Elastic design is a non-linear programming problem because the equations of equilibrium are non-linear. These equations for trusses has the form

$$f(\sigma_i \; A_i) = P_i \qquad .. \; 1.1$$

where both the member stress 'σ_i' and the area of the member 'A_i' are unknowns and they occur in pairs making it a non-linear set of equations. 'P_i' is the external load vector. This type of design where member parameters and member

stresses are determined to minimize the volume (also called the objective function in the literature) is the direct design of structures.

The above design is based on the stress criterion. There is another type of design in which the basis is the ultimate strength of the structure. This approach goes under the name Limit design or Plastic design of structures. Like a direct elastic design there is also a direct plastic design of structures. The plastic design of structures comes under linear programming.

Although the direct design is an excellent method the labour involved is extremely high and with the present computation facility only small structures like a two storey portal frame, gables, small trusses with only a few panels are attempted.

Since contemporary structures are highly complicated and are unusually large like a hundred storey building or a long truss bridge, the above direct design with the present knowledge and computation facility becomes highly involved as the number of equations of analysis and constraints of design are unusually large. Much work is being currently carried out in direct design of structures. F. Moses (1)* has transformed the non-linear elastic design of a truss to a step by step linear

*Figure in brackets refers to reference number in the Bibliography.

design problem by Taylor series expansion. L.C. Schmidt (2) has developed methods for the direct elastic design changing even the geometry of the structure. Bigelow (3) has worked on the linear programming problem applied to plastic design. Other pioneers on this field are Prager (4), Brown (5), Save (6) and Koopman (7).

In contrast to the direct design there is also the indirect design also called the interative design of structures. In this method first the analysis is completed and the elements are proportioned according to the design constraints. The cycle of analysis and design are repeated until the sections properties remain constant i.e. the iteration on design converges.

Malcolm (8) puts the above iterative design in a sequential form as:

(1) Estimation (i.e. guessing) of the member sizes.

(2) Analysis for stress and deformation for the above proportions.

(3) Checking of the sections for strength stability and serviceability for the forces of the above analysis.

(4) Successive modifications of the original guess for member sizes until convergence in the member dimension is achieved.

For a clear understanding of the principles underlining the four steps of an iterative design as suggested by Malcolm the second step is treated first.

1.2 Stress and Deformation Analysis of a Structure

This is the most important and involved procedure in the iterative design of structures. This is concerned with the evaluation of forces and deformation at every point of the structure.

There are two fundamental methods of analysis, the method in which the laws of equilibrium are applied first leads to the equations of geometric compatibility and is known as the Compatibility or the Force or the Flexibility method and the method in which the conditions of compatibilities are used first gives rise to the equations of joint equilibrium and is called the Equilibrium or the Displacements or the Stiffness method.

Both the methods finally lead to a set of simultaneous equations which may be the compatibility or the Equilibrium equations.

Although any of the methods can be used for the analysis of any structur, it is obvious that for a given structure the method which is less involved should be chosen.

1.3 Force Method

For hand calculation Flexibility method is less involved and is more widely used for structures with low degree of statical indeterminacy, as the set of equations to be solved are less in number. The persons associated with this method are Argyris (9) and P.B. Morice (10) although the method was first

formulated in 1888. The method of consistent deformations and all Energy approaches (11) of analysing structures come under this method. Basically this method is the same as the method of Mohr (12) who formulated the basic equations and called them as the simultaneous condition equations of an indeterminate structure which expresses the geometric coherence of the structure (i.e. the geometric compatibility of the structure).

Up to the present time Force method was popular but in the machine computation this method is not generally used, although for trusses the method yield very few equations, because the process of generating these equations in a computer involve two major difficulties:

(1) In the Force method to generate the equations a stable released structure is necessary which is to be obtained out of the real structure by removing all the redundants. For a complex structure the determination of the degress of redundancies and a choice of a released structure which may save a lot of computation time is extremely difficult as this is a structure dependent phenomenon and varies from structure to structure.

(2) The compatibility equations are the equations of deformations. Even when all the member forces are known to calculate the deformations and hence to generate each compatibility equation of the Force method all the members of the

structure have to be scanned. For hand calculations many short-cuts may be resorted to, to simplify the above generation of the final equations depending upon the structure. Since a computer programme is generally written for all types of structures such labour and computation saving is not possible which make the computation tedious and time consuming. This difficulty for future reference will be known as the structure wide effect of Force method.

In short, Force method is generally not recommended because of the difficulties involved for the generation of the equations, although the number of these equations for low indeterminate structures are very less. For example, for a truss with two redundants and 50 joints such equations are only two whereas for Displacement method the number may be 100 or 150 according to the dimensions (plane or space) of the truss.

1.4 Displacement Method

The Displacement method of structural analysis is basically due to Maney (13) who published the slope deflection equations in 1915. The number of equations grow rapidly as the structures gets complicated, for example a four storey and four bay plane building frame considered to be a very small structure, gives rise to 32 simultaneous equations considering only two displacements (joint rotation and joint translations) for each of its 16 joints. It is extremely difficult to solve the 32 equations by hand. Hence before the introduction of calculating machines various relaxation and

iterative procedures are developed to solve the structures by hand. Few of the notable methods for analysing structures by hand are given below.

1. Moment Distribution

This is a relaxation method and is due to Hardy Cross who published this in 1930. In this method the building frame is held against sway at each floor level and joint equilibriums are satisfied by distributing the unbalanced moments among all the members meeting at the joint according to their stiffnesses. Storeys are then allowed to sway.and member end momentsdetemined for each sway. These moments are balanced and storey shear equations wiitten for each storey which gives the sway. By superposing the solution of unsway and sway cases the final moments are calculated. This method is unsuitable for multi-storeyed frames as the number of moment distributions increases according to the number of storeys but are powerful for non-translatory structures.

2. Kanis Method (14)

G. Kani developed an iteration method to solve the slope-deflection equations. In this method both non-sway equations and storey shear equations are balanced together. This is an excellent contribution but limited to building frames with constant storey height for a storey.

3. Takabeya Method (15)

Also known as the Japanese method iterates

the slope deflection equations at both far end and near end simultaneously for a few cycles and then distributes the unbalanced moment at a particular joint according to the principle of moment distribution. This seems to be a powerful method as Takabeya has solved a two hundred storey and twenty-seven bay building frame by hand within seventy-eight hours.

The complexity of modern structures and the use of digital computer has discarded the above methods. The equations of the Displacement method are written in the matrix form and is the most popular method in machine computation. It does not have any of the difficulties encountered in the Force method. Each equation of this method deal only with the number of members meeting at a joint and are are completely localized to the joint and the for end joints of the member meeting at the joint. This gives rise to a regular band for most of the symmetric and regular structures. Since redundants of the structures are not needed at any phase of the analysis the difficulty of determining a basic structure necessary in the Force method is avoided. The popularity of this method is due to the above two advantages, although the final equations of Displacement method for trusses are sometimes unusually large than the equations of the Force method.

There are two ways of arriving at the final equations of the Displacement method. The first method is

based on the principle of virtual work and involves the multiplication of a number of matrices the dimensions of which are much larger (7 to 8 times larger) than the dimension of the final equations and is seldom used in the computers. Semih S. Tezcan (16) and Pestal (17) use this method to arrive at the final equations. Semih S. Tezcan (16) assumes the sway of a storey constant i.e. the axial defomations are neglected where as Pestel (17) includes axial deformations of beams and columns. In the other method joint equations are directly written and is known as the method of merging.

In 1962 Eiseman and Namyet (18) published the above approach for space frames. They have generalized the slope-deflection equations for discrete space structures. They have developed a computer programme called FRAN (19) (FRame ANalysis) in which the total structure is divided into several units and eliminating some of the joint displacements while proceeding from one unit to other they are able to solve huge structures in computers using external storage.

Semih S. Tezcan (20) has also used the generalised slope deflection equations to arrive at the final equations.in matrix form. Then breaking up the structure stiffness matrix, into two trangular matrices and using Cholesky's square root scheme, has shown that without the help of external storage, 1000 equations with a band width of 27 can be solved by a IBM 7090 computer having 32000 word capacity which othersise

can invert only a 150 x 150 matrix. With the help of external storage he has handled as high as 3500 equations. An another paper Tezcan (21) has formulated the joint equilibrium equations using slope objlection equations andneglecting the axial deformations. Here he has formulated the joint and storey equations separately.

R.W. Clough and Ian. P. King (22) have developed a programme in which the floor displacements are grouped together, thereby arriving at a three band stiffness matrix for each frame of the building. The effect of shear walls in between two girders are taken in formulating the slope deflection equations and they have solved the final matrix by partitioning technique.

J.E. Goldberg (23) has contributed a method to calculate stresses due to lateral loads in multi-storeyed building frames. He has modified the slope deflection equations to take into account shear stiffners in the form of walls or shafts. The equations are generated from top floor of the building and while proceeding to the bottom most floor, the displacements of the top floors are eliminated and where the bottom most floor is reached the equations contain only the displacements of this floor. This displacements are determined and then the building is scanned from bottom to top to yield the total displacement matrix.

Other people associated with this method are

Archer (24), Benscotter (25), Denke (26), Klein (27), Martin (28), Turner (29) ad et.

~~The~~ Basically most of the works deal with two fundamental procedures:

1. The Model:

Here the structure is idealised namely - shear wall are introduced, building is visualized as a space, tier or a plane frame. Trusses are introduced to represent the slabs and walls and all types of improvement is thoughout of to represent the real structure by a model which may resemble the actual structure closely.

2. The Computation:

The above model leads to a set of equations and the researcher developed means of accommodating and solving these equations with the present digital computers having a limited memory.

Most of the modern structures are very complex in nature and which need not give a banded set of equations like rings and space trusses. To solve these structures the above procedure is sometime quite involved with the present computation facility. For such structure an iterative technique is developed. In this procedure joint stiffness matrix is generated and sisplacements of the joint is calculated. The procedure is repeated for all joints several times until

the displacements have a stationary value. This type of solving the structures saves the memory of a computer as the only storage necessary is the joint stiffness matrix. To accelerate the convergancy initial values of joint displacements are calculated by approximate methods. This is similar to Gauss Seidel method in which only the required number of equations necessary are generated and stored. This method is applicable to any type of irregular structure and there is no need for a banded stiffness matrix.

Once the analysis is complete next step is the design of a structure. As already stated the most important factor of a design is the economy. Since the structures dealt are rigid jointed and pin-jointed structures and each need different treatment they are taken up separately.

1.5 Pin-jointed Structures

In 1900 Cilley (30) pointed out that for an indeterminate truss under one load condition, the minimum weight structure is a statically determinate structure in which the cross-section dimension of each member is selected to achieve the allowable stress for that member. For multiple load condition he used the concept of a fully-stressed design. A fully-stressed design is defined as one in which each member reaches its allowable stress under at least one of the load system. In 1954 G. Sved (31) also proved that under one load condition the minimum weight truss is a determinate one.

In 1958 L.C. Schmidt (32) worked on the indeterminate truss and noted that there are several fully-stressed designs for a truss under the load conditions and the least weight fully-stressed structure is the minimum weight structure. But in 1960 L.A. Schmit (33) solved the above problem using mathematical programming and came to the conclusion that minimum weight structures are not necessarily fully-stressed designs.

In 1964 R. Razani (34) examined the relationship of the minimum weight design and the fully-stressed design and concluded that fully-stressed design need not converge to a minimum weight design. In 1966 William Weaver (35) developed an automatic design procedure for space trusses. This is a fully-stressed design and also includes dynamic effects like the resonant frequency requirements.

Most of the above structures have a fixed geometry. Schmit extended the concept of overall geometric configuration as a design parameter. A direct design, to evaluate the geometric configuration to carry the load was taken up by Michell (36) in 1904. His structures are known as Michell structures, the geometry of these structures are such that they are not practically usable. But the theorems and procedure of Michell is extensively used in the aircraft industry.

In this thesis minimum weight structures are

designed by an iterative procedure. The concept of a fully-stressed design for one load condition is explained in the Chapter 'Feasibility of a Structural Design'. It is proved that for an indeterminate structure with one load condition a full-stress design is not at all possible as it violates the fundamental compatibility equations of a pin-jointed structure. It is also shown that even for multiple load condition, a fully stressed design may not be possible. The design procedure for these structures are dealt in this Chapter and in the section 4.13 of Chapter IV.

1.6 Rigid-jointed Structures

If the research on the analysis of such structures are compared with the research on design it can be clearly seen that analysis rather than design has consumed far greater share of attention although analysis is only a part of the design procedure. Sophisticated computer programmes are developed for the analysis of discrete and continuous structures but for design one depends upon the age old methods of cut and try procedure.

In 1939 Grinter (37) pointed out this fact and introduced a concept of automatic design of beams and frames. In this procedure he tried to distribute the fixed end modulus instead of the fixed end moments in the moment distribution procedure. At intermediate steps he charged the distribution factors so that at the end of the distribution the balanced

modulus and the actual modulus agreedwithin a short margin. He has also shown the procedure to achieve economy. This concept is difficult to use for modern complex structure for the same reasons which makes moment distribution obsolete.

Dhanjoo N. Ghista (38) also tried to design a full stressed structure, every section of which must reach its maximum allowable stress at some times during the application of alternate load systems. Obviously the cross-section is not kept constant. Initially the forces and moments in the structure are assumed to satisfy the conditions of equilibrium between the external and internal forces and design parameters are obtained for full-stress conditioned. These do not satisfy the compatibility conditions and section properties are successively modified to satisfy compatibility. He could only solve two span beams and also recommended the method for portal frames only. Dhanjoo N. Ghista (39) has also tried in another scheme to determine optimum frameworks under different load systems using the theorems of Mitchell and Maxwell. These methods along with mathematical programming methods are only used for small structures.

A 100-storey building is designed by Fazlur R. Khan (40) at ed. Here mebbers are proportioned by a IBM 1620 and preliminary data for member sizes determined which were fed to a bigger computer and the structure was analysed

as a space structure. Louis A. Hill (41) designed a building frame using electronic computers. This was completely automatic beginning from initial sizing and dead load calculation to the final design. Elements were designed for optimum cost, taking into account the cost of concrete and forming.

A language STRUDL (42) (STRUctural Design Language) is developed for the design of structures. This language has the following operations:

(1) Analysis Operation

It can do preliminary and determinate analysis of a structure and then the stiffness and dynamic analysis of a structure.

(2) Design Operation

It can select member sizes and check these sections according to some codes of Specification.

(3) Detailing Operation

It can determine weight cost and can detail components of a structure.

Kinra and Fenves (43) worked on a computer aided design. They analysed and checked concrete structures. The type of structure studied by them is flat plate reinforced concrete buildings. The slab has a uniform thickness so also the girders and beams which they designed according to ACI Code. The slab is idealised as girders (Tee beams) for the elastic analysis. All components are designed for both working stress and ultimate strength.

Beck (44) has developed a programme to design columns of multi-storeyed buildings subjected to bending about both the axis. In this programme he determines equilalent moments and for a given section satisfies the interaction formula.

In this thesis the design of plane frame. is completely automated. The frame is analysed by the point iteration technique similar to that explained for trusses. Initial member parameters are determined by approximate analysis. Contilever method is adopted for the analysis of Wind Load and for gravity load beams are assumed pined to the columns to determine the column loads and completely fixed for beam moments.

An uniform I-section is designed for columns and beams. Since at the economic depth of I-sections it has the minimum area, this area is achieved as follows: For a particular depth the section is designed to satisfy the interaction relation. Then the depth is varied and the process repeated until the area of the section is a minimum. In short for each member an economic section, uniform throughout its span is designed to resist the forces it is subjected to. Then the structure is analysed for the above member properties until the sections properties converge. Since the frame is made up of elements and elements designed are economical ones the frame is also an optimum one.

The thesis contains the following chapters:

Chapter II - Analysis of Structures:

The structures namely pin-jointed plane and space trusses and plane frames are analysed by point iteration of the stiffness equation. Computer programmes are developed which can analyse Beams, Plane Frames, Gables, Arches, Plane and Space Trusses.

Chapter III - Feasibility of a Structural Design:

In this section the fundamental equations of analysis are examined for both pin-jointed and rigid jointed structures. The feasibility of a full stress design for both cases are examined.

Chapter IV - Desgin of Structures:

In this chapter the preliminary analysis and structural design for both the type of structures are treated and suitable computer programmes are developed.

Chapter V - Illustrations:

Here several examples are illustrated.

Chapter VI - Conclusions and Discussions:

Conclusions and future work on the field is suggested.

CHAPTER II

STRESS AND DEFORMATION ANALYSIS OF STRUCTURES

A structural analysis is concerned with the determination of forces and deformations at every point of the structure. In case of discrete structures like frames and trusses, interest is largely centred on the displacements of the joints and internal forces at the member ends. The joint deformations and member end forces establishedthe complete stress and deformation pattern in each member of a discrete structure. Once these forces and deformations have been found, the detailed calculation of the conditions at internal points of a member depends upon the characteristics of the member. The material of the members are assumed linearly elastic. For this type of structural members the internal forces and deformations are linear functions of the joint and members end actions. So for discrete structures the analysis may be regarded as complete when the joint displacements and member end forces are determined. The deformations and forces in a linear structure are interrelated and from one the other can be easily calculated. Accordingly there are two basic methods of analysis, The Force method and the Displacement method.

In the Force method forces are taken as the unknowns and member deformations are calculated. The displacement compatibility equations are satisfied from which unknown forces are evaluated. These are generally the redundant reactions. After this the stress or strain analysis of the entire structure is completed by the laws of statics and the stress strain relations.

In the Displacement method joint deformations are taken as the unknowns, member forces are calculated and joint equilibrium is satisfied from which joint deformations are determined. After this using statics and force deformation relationships the whole stress analysis of the structure is completed.

Each method has its own merits. The Displacement method is more amenable to digital computers and hence this method is adopted. The final equations of the displacement method can be arrived at by several ways. Here slope deflection method is used to arrive at the equations of the Displacement method.

2.1 Formulation of Slope Deflection Equations

In this method member end deformations (the slopes and deflections) are the unknowns. The equations establish a relation which expresses the end forces of an elastic member as a function of end deformations. This method may be used for the analysis of statically determinate or indeterminate frames or trusses.

Here the equations are developed for a linearly elastic straight member having constant cross-section throughout its length. It is further assumed that the cross-section is doubly symmetric i.e. the centroid and the shear centre coincide resulting in no twisting moments for transverse loads acting through the centroidal axis.

Each member is referred to an individual coordinate system. In this system the x-axis (also the axis-3) is taken along the centroidal axis of the member, y (also the axis 2) and z axis are along the principal axes of inertia. This coordinate system is an orthogonal right-handed system and hence forth be referred to as the local system.

2.2 Sign Convention

In the local system, forces and deformations are considered positive at a joint along positive coordinate axis.

2.3 Plane Frame Element

For such an element there are three displacements along each member ends namely the displacements, u, along x axis and v along y axis and the rotation, θ, about the z axis and a compatible set of forces.

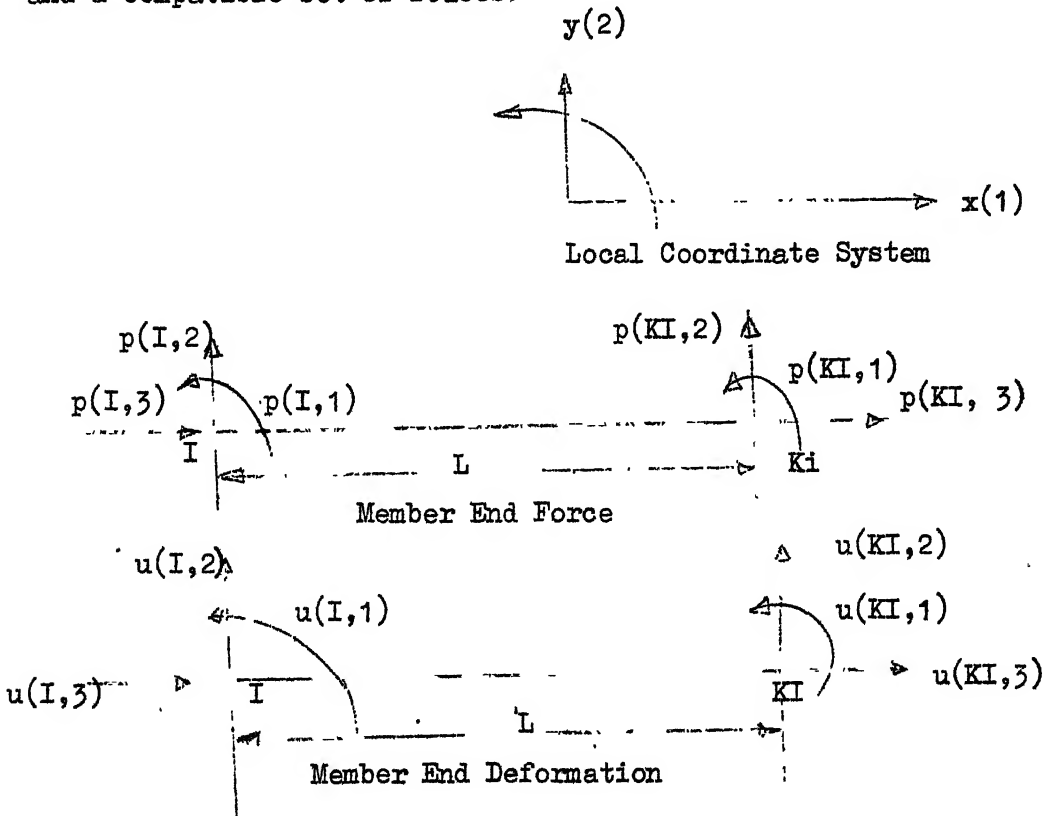


Fig 2.1

2.4 Nomenclature

A member is designated as I and KI, where I refers to the left end and KI to the right end of the member. p(L,M), u(L,M) denote the force and deformation at the end L and towards (or about in case of couple and rotation) M axis respectively. The force and deformation vectors are respectively

$$\begin{bmatrix} p(I,1) \\ p(I,2) \\ p(I,3) \end{bmatrix} \begin{array}{l} \text{Moment positive about z-axis} \\ \text{Shear force positive along y-axis} \\ \text{Axial force positive along x-axis} \end{array}$$

$$\begin{bmatrix} u(I,1) \\ u(I,2) \\ u(I,3) \end{bmatrix} \begin{array}{l} \text{Rotation positive about z-axis} \\ \text{Sway deformation positive along y-axis} \\ \text{Axial deformation positive along x-axis} \end{array}$$

2.5 The slope deflection equations in the local system are:

$$p(I,1) = 2EI/L \quad (2u(I,1)+u(KI,1)) - 6EI/L(u(KI,2) - u(I,2))$$

$$p(I,2) = 6EI/L^2 \quad (u(I,1) + u(KI,1)) - 12EI/L^3(4(KI,2) - u(I,2))$$

$$p(I,3) = -AE/L \quad (u(KI,3)-u(I,3)) \qquad .. \ 2.1$$

I, A, E and L refer respectively to the moment of inertia along the bending axis, the area of member, the Youngs modulus of the material and span of the member.

Equation 2.1 in the matrix notation may be written as:

$$\begin{bmatrix} p(I,1) \\ p(I,2) \\ p(I,3) \end{bmatrix} = \begin{bmatrix} 4EI/L & 6EI/L^2 & 0 \\ 6EI/L^2 & 12EI/L^3 & 0 \\ 0 & 0 & AE/L \end{bmatrix} \begin{bmatrix} u(I,1) \\ u(I,2) \\ u(I,3) \end{bmatrix} + \begin{bmatrix} 2EI/L & -6EI/L^2 & 0 \\ 6EI/L^2 & -12EI/L^3 & 0 \\ 0 & 0 & -AE/L \end{bmatrix} \begin{bmatrix} u(KI,1) \\ u(KI,2) \\ u(KI,3) \end{bmatrix}$$

In a condensed form for further work the above equation may be written as:

$$p(I) = K(I,J)\ D(I) + K(J,I)\ D(J) \qquad .. \ 2.2$$

where

$$p(I) = \begin{bmatrix} p(I,1) \\ p(I,2) \\ p(I,3) \end{bmatrix} \quad D(I) = \begin{bmatrix} u(I,1) \\ u(I,2) \\ u(I,3) \end{bmatrix} \quad D(J) = \begin{bmatrix} u(KI,1) \\ u(KI,2) \\ u(KI,3) \end{bmatrix}$$

$$K(I,J) = \begin{bmatrix} 4EI/L^2 & 6EI/L^2 & 0 \\ 6EI/L^2 & 12EI/L^3 & 0 \\ 0 & 0 & AE/L \end{bmatrix} \quad K(J,I) = \begin{bmatrix} 2EI/L & -6EI/L^2 & 0 \\ 6EI/L^2 & -12/EI/L^3 & 0 \\ 0 & 0 & -AE/L \end{bmatrix}$$

2.6 Truss Element

For a pin jointed truss element may it be plane truss or space truss the members can carry only axial forces, but each joint can have either two or three displacements along the coordinate axes depending on whether the truss is of the plane type or space type. There is no elastic bending of member. In the local system only axial displacement (i.e. along centroidal axis) i.e. u(I,3) and u(KI,3) create forces in the member other displacements do not create member forces. Because of this the slope deflection equation for plane or space truss in the local system has the form:

$$p(I,3) = -\ AE/L(u(KI,3) - u(I,3))$$

$$p(I,2) = p(I,1) = 0$$

$$.. \ 2.3$$

In the condensed form for future work it may be written as:

$$p(I) = K(I,J)\ D(I) + K(J,I)\ D(J) \qquad .. \ 2.4$$

where $p(I) = p(I,3)$; $D(I) = u(I,3)$; $D(J) = u(KI,3)$;

$$K(I,J) = AE/L;\ K(J,I) = -AE/L.$$

Equation 2.4 is the slope deflection equation for a plane or space truss element in the local system.

2.7 Coordinate Transformation

The above slope deflection equations are with reference to the local system. There are as many local systems as there are members. In order to formulate the equations of equilibrium all the above equations have to be referred to a common coordinate system. Such a system will henceforth be called as the Global System. This is also an orthogonal rectangular system.

In this section local variables will be transferred to global variables through a transformation matrix.

Let x be a vector of local variables (may be the deformations or the forces in the local system) and X be the same variables in the Global System. Let T be the matrix that transforms the local variables to the global variables and be defined as:

$$x = T\ X \qquad \text{.. } 2.5$$

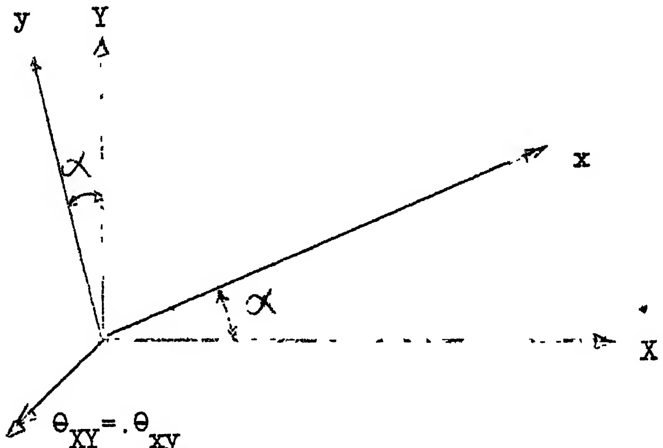
T is known as the transformation matrix.

When one orthogonal set is transformed to another orthogonal set by rotation, the transformation is said to be orthogonal. For such a transformation (as is the case here)

the matrix T can be shown to have the property:

$T^{-1} = T^{T}$ (Inverse of 'T' matrix = Transpose of 'T' matrix)

2.8 Transformation Matrix for a Plane Frame Element



The vectors to be transformed are the deformations and forces.

Let local vector be (θ_{xy} x y)

Similar Global vector be (θ_{XY} X Y)

Since Z axis does not change θ_{xy} is same in global or local system:

$$x = X\,\text{Cos}\,\alpha + Y\,\text{Sin}\,\alpha$$

$$y = X\,\text{Sin}\,\alpha - Y\,\text{Cos}\,\alpha$$

$$\theta_{xy} = \theta_{XY}$$

In matrix notation:

$$\begin{bmatrix} \theta_{xy} \\ y \\ x \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\text{Cos}\,\alpha & \text{Sin}\,\alpha \\ 0 & \text{Sin}\,\alpha & \text{Cos}\,\alpha \end{bmatrix} \begin{bmatrix} \theta_{XY} \\ Y \\ X \end{bmatrix} \quad \text{or } (T) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\text{Cos}\,\alpha & \text{Sin}\,\alpha \\ 0 & \text{Sin}\,\alpha & \text{Cos}\,\alpha \end{bmatrix}$$

2.8 Transformation Matrix for Plane Truss Element

It is same as plane frame element except that there is no θ_{xy} in this case.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} ; \quad T_1 = \begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} ; \quad T_1^{-1} = \begin{bmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{bmatrix}$$

But since in local system we need only variables in the x-direction only we shall only be using a part of the transformation matrix namely;

$$x = (\cos\alpha \quad \sin\alpha) \begin{bmatrix} X \\ Y \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix} (x)$$

Henceforth we shall define

$$T = (\cos\alpha \quad \sin\alpha) \text{ and } T^{-1} = T^T = \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix}$$

For the purpose of calculation T-matrix is always used and because of the above equations transpose of T is taken as its inverse although it has its origin with T_1 matrix.

2.10 Transformation Matrix for a Space Truss Element

It is similar to plane truss element except that the deformations or forces in Z direction (in global system) has to be considered.

In this case the transformation matrix has the form

$$T = (l_x \; l_y \; l_z)$$

$$T^T = T^{-1} = \begin{bmatrix} l_x \\ l_y \\ l_z \end{bmatrix}$$

where l_x, l_y, l_z are the direction cosines of the local x-axis in the global system.

It may be noted that in the local system force deformation equations of plane frame or truss element has the form:

$$p(I) = K(I,J)\ D(I) + K(J,I)\ D(J) \qquad \text{.. } 2.6$$

where each variable refer to a particular matrix as the case may be. The transformation matrix also transforms p(I), D(I), D(J) from local to global system and has particular form for the particular case. The general case taking equation 2.6 and proper T-matrix will be dealt with. The particular cases may be arrived at by proper substitutions.

2.11 Equivalent Joint Loads

A structure may be subjected to loads at the joints and loads along the member (only in case of plane frame elements). For forces acting in the span of the member, equilibrating joint forces (Fixed End reactions) are calculated. These loads when applied in the opposite sense form the equivalent joint loads in the local system. Local joint forces are transformed into the global forces and then added to the loads applied to the joints directly to form the total joint load. In case of truss members this case does not arisesas there are no member forces. For a plane frame

$$\text{Let} \quad \begin{bmatrix} S(I,1) \\ S(I,2) \\ S(I,3) \end{bmatrix} \quad \begin{array}{l} \text{be the Moment} \\ \text{force in x and} \\ \text{y direction respectively at a joint} \end{array}$$

Let $\begin{bmatrix} M(I,1) \\ M(I,2) \\ M(I,3) \end{bmatrix}$ be the corresponding fixed end reactions due to member forces.

The total Joint Force is given by

$$\begin{bmatrix} F(I,1) \\ F(I,2) \\ F(I,3) \end{bmatrix} = \begin{bmatrix} S(I,1) \\ S(I,2) \\ S(I,3) \end{bmatrix} - \begin{bmatrix} T^T \end{bmatrix} + \begin{bmatrix} M(I,1) \\ M(I,2) \\ M(I,3) \end{bmatrix} \qquad .. \ 2.7$$

2.12 Joint Equilibrium

The internal forces in the structure and the external loads must satisfy the conditions of equilibrium. As stated earlier for a discrete structure the equilibrium is satisfied when the joint equilibrium is established. For this the force-deformation relation is transferred to the global system and equilibrium equation at any joint is obtained by summing up the forces both internal and external along the axis of the global system, to zero.

$$\sum F(I,1) = 0 \quad \sum F(I,2) = 0 \quad \sum F(I,3) = 0 \qquad .. \ 2.8$$

$$d(I) = T\ D(I); \ d(J) = T\ D(J) \quad p(I) = T\ P(I) \qquad .. \ 2.9$$

Here variables in capitals refer to global system and variable in small case letters refer to local system.

Substituting for local quantities in equation 2.6

$$P(I) = T^T\ K(I,J)\ T\ D(I) + T^T\ K(J,I)\ T\ D(J). \qquad .. \ 2.10$$

Substituting equation 2.10 in equation 2.8

$$F(I) = \left(\sum T^T\ K(I,J)\ T\right) D(I) + \sum T^T\ K(J,I)\ T\ \ D(J) \qquad .. \ 2.11$$

2.11 is the basic joint equilibrium equations.

The summation A extends over all the members meeting at the Joint 'I'; D(I) is the same for all members meeting at that joint where as D(J) differ from member to member.

The equation 2.11 is a general equation and holds good for both plane, space trusses and plane frames In case of a plane truss it is a 2 x 2 matrix equation and for both space truss and plane frame it is a 3 x 3 matrix equation.

One equation of the type 2.11 is written for each joint giving rise to the main system of equations, to be solved for the joint displacements.

2.13 Boundary Equations

It may so happen that for a boundary the joints may be restrained in one or more direction. A general discussion is given below for such joints.

Let the Ith joint be restrained along 2nd direction (i.e. in the direction of sway displacement). For this case:

$$u(I,2) = 0; \quad P(I,2) = 0$$

Let $\begin{bmatrix} a \end{bmatrix} = \sum T^T K(I,J)\ T$ and $\begin{bmatrix} b \end{bmatrix} = \sum T^T K(J,I)\ T\ D$

Then the equation 2.11 may be written as:

$$\begin{bmatrix} P(I,1) \\ P(I,2) \\ P(I,3) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} U(I,1) \\ U(I,2) \\ U(I,3) \end{bmatrix} + \begin{bmatrix} b(I,1) \\ b(I,2) \\ b(I,3) \end{bmatrix}$$

Since $P(I,2) = 0$, this can be achieved by setting $a_{21} = a_{22} = 0$ $a_{23} = b_{12} = 0$ and $U(I,2) = 0$, can be achieved by setting $a_{12} = a_{22} = a_{32} = 0$. That is to say delete the proper rows and columns as shown and rewriting the matrix equation for the restrained joint we have:

$$\begin{bmatrix} P(I,1) \\ P(I,3) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{bmatrix} \begin{bmatrix} U(I,1) \\ U(I,3) \end{bmatrix} + \begin{bmatrix} b(I,1) \\ b(I,3) \end{bmatrix} \quad .. \; 2.13$$

Equations similar to 2.13 may be written for semi-restrained joints. If there are two restraints delete two rows and two columns. For joint which is restrained in all direction as can be seen there exist no equation for that joint.

2.14 Evaluation of Member End Forces

Once the joint displacements are obtained by solving the set of linear simultaneous equations the member end forces can be determined by using the slope deflection equations. Since this equations are in the local coordinates all global variables are to be transformed to local variables. The only variable needed is the displacements and can be found by relation 2.9.

$$d(I) = T\ D(I); \quad d(J) = T\ D(J)$$

Substituting in equation 2.4

$$p(I) = K(I,J)\ T\ D(I) + K(J,I)\ T\ D(J) \quad .. \; 2.14$$

Equation 2.14 gives the member end forces in the local coordinate system.

2.15 Method of Solution

There are two methods for solving the linear simultaneous equations (2.11) for the joint displacements.

1. Direct Method

2. Iterative Method

1. Direct Method

In this case equations are written for all the joints. These equations in matrix notation have the form:

$$F = K D$$

where F and D are respectively the joint force and joint deformation vectors. Both are $(L \times N + M) \times 1$ - matrices

K is the coefficient matrix (coefficient of joint displacements) also known as the stiffness matrix and it is a $(L \times N + M) \times (L \times N + M)$ matrix; where L is the total number of fully deformable joints and N denotes the number of possible joint displacements; L is equal to 3 for both space truss and plane frame but is 2 for a plane truss.

M: Number of free displacements of all restraint joints (generally the boundary joints).

2.16 Illustration

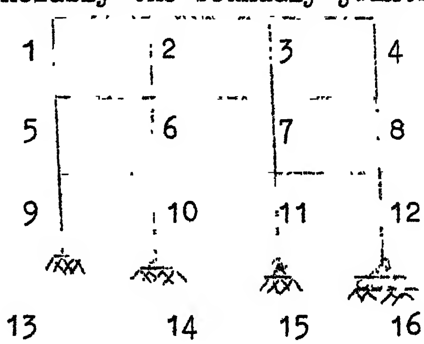
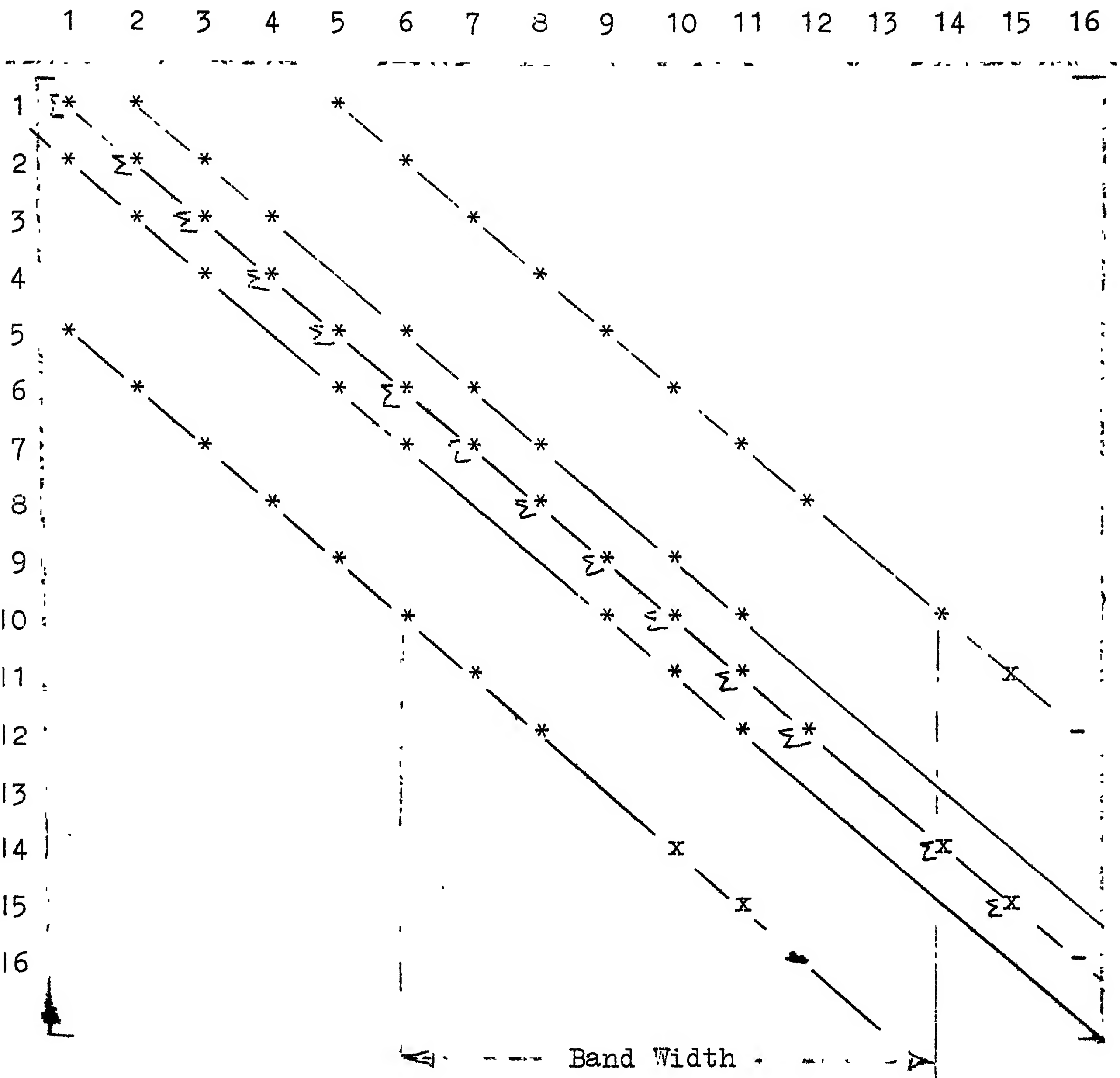


Fig. 2.2

VIEW OF THE STIFFNESS MATRIX OF THE STRUCTURES SHOWN IN FIG.2.2



Five Band Stiffness Matrix

Dimension of 'D' matrix = 3 x (12) + (0 + 1 + 1) 2) = 40.

Hence dimension of 'K' matrix is 40 x 40.

Joint equations: 1 through 12.

Here each joint has three equations. The equation for joint 1 involves only the deformations of joint 1 along with that of 2 and 5 only. The joints 2 and 5 will be referred to as the far ends and joint 1 itself as the near end joint. It can be easily seen that any joint equation involve all far end joints deformations and the deformation of the joint in question. The maximum number of joint deformations among all the equations equals the number of bands in the 'K' matrix and the spread of all bands taken together is referred to as the band width.

For the above structure by inspection the number of bands equal 5 and band width equals 3 x 9 = 27.

The figure F.3 gives the total stiffness matrix. Where a star (*) represent a 3 x 3 matrix cross (x) 2 x 2 matrix and dash (-) one by one matrix.

Once 'K' matrix is generated the displacements can be obtained as $D = K^{-1} F$.

Direct inversion is only possible for relatively small structures as the computer exhusts all memory for bigger ones. This is because lot many null-matrices are to be stored. Hence this method is not suitable for large structures having many degrees of freedom.

2. Iterative Method

Advantage could be taken of the null matrices and storage can be considerably reduced by working with partitional matrices. In this case the total stiffness matrix 'K' is generated and stored and can be solved by several methods like guss Seidel, Jacobi iteration etc. In this case economy in storage may be achieved by storing only the bands provided it has a regular band pattern. It may be noted that for irregular structures and closed structures we do not have regular bands. In such cases storage economy cannot be easily achieved. For such cases the method developed below is helpful.

Point Iterations

In this method one goes from one joint to another. At every joint the particular joint equation 2.11 is generated and that joint deformation is calculated. The procedure is repeated for all joints several times until the iteration is stationary. This is similar to Gauss Seidel method.

$$P(I) = T^T K(I,J)\ T\ D(I) + T^T K(J,I)\ T\ D(J)$$

$$T^T K(I,J)\ T\ D(I) = P(I) - T^T K(J,I)\ T\ D(J)$$

Let $A = T^T K(I,J)\ T$; $B = T^T K(J,I)\ T$

$$A\ D(I) = P(I) - B\ D(J)$$

where both A, B, D(I), P(I) and D(J) are matrices.

$$D(I) = A^{-1}\ (P(I) - B\ D(J)) \qquad \text{.. 2.12}$$

Procedure:

a) Number the joints in an optimal fashion.

b) Let JJ be the number of members meeting at a joint.

c) For each member meeting at a joint generate T, K(I,J) and K(J,I) - matrices

d) For first cycle assume all joint displacements (may be assumed zero to start with).

e) Calculate T^T K(I,J) T and T^T K(J,I) T D(J). Store them in matrices DENOM and UPPER respectively.

f) Determine the inverse of DENOM

g) Premultiply the Inverse of DENOM to (P(I)-UPPER) thereby obtaining the displacements D(I). Repeat the process for all joints. For second and subsequent cycles use the displacements of the previous cycles and repeat for all joints several times until the joint displacements have stationary values.

In fact this is the Gauss Seidel method. The convergency is quite fast. The convergency may still be accelerated by using overrelaxation factors. In this analysis the new displacements are obtained as:

$$D(K) = D(K) + \beta(D(K) - D(K-1)) \qquad .. \ 2.13$$

where K refers to the iteration cycle and β the over-relaxation factor. According to the above equation for the next iterative cycle the displacements of the previous cycle may be changed by a multiple of the difference between the displacements of two cycles.

The value of β generally lies between 0 and 1 and a value of $= 0.9$ suits the framed structures.

It is important to note that for the seocnd design cycle the first cycles' deformations are taken and since the stiffness of the structure does not change radically only two to three more cycles for analysis may be enough to make the iteration stationary.

The iteration is said to be stationary when the equilibrium of the structure is satisfied. For this the total unbalanced load for each joint is determined. This is added up for all the joints to achieve the total unbalanced loads for the structure. The iteration is said to be stationary when this total unbalanced load of the structure is negligibly small i.e. 2 to 5% of the external load.

CHAPTER III

THE FEASIBILITY OF A STRUCTURAL DESIGN

The feasibility of the economic design of trusses and plane frames will be considered. The problem here is, given the geometric configuration of a structure, subjected to a set of loads to determine the design parameters to achieve an economic design.

Any valid design or analysis must satisfy the following fundamental equations.

1. Stress-strain Relation

The structures designed are assumed to be in the elastic range. In the elastic range the stress-strain relationship is simple and involves a one to one correspondence of stress and strain. This means the stress and strain relation is independent dent of the loads the member has to carry, i.e. the displacements and internal forces are linear functions of the external loads. Such a structure is called a linear structure. Since the changes in the geometry between the undeformed and deformed structure for a frame or a truss are really small most of these structures under working loads behave approximately as linear structures. Because of this fact the stiffness and flexibility matrices are independent of the applied loads and the principles of superposition holds good.

3.2 Equilibrium Equations

The forces (i.e. the reactions) at the ends of each member must be such as to keep the member in equilibrium. The reactions at the boundaries of the structure must be such as to keep the structure as a whole in equilibrium. The sum of the reactions acting at any joint must be equal to the external loads applied at the joint. This gives rise to the joint equilibrium,which are the main system of equations of the stiffness method of analysis as indicated in the earlier Chapter.

3.3 The Conditions of Compatibility

These refer to the displacement compatibility equations although the above set of equilibrium equations may be called as the compatibility of forces. According to this the displacements of each of the member must be compatible with the displacements of the joints to which the member is attached or in other words the members so deform that they can all fit together. These equations in the displacement method is written as:

$$v = Td \qquad .. \ 3.1$$

where d is the number of independent joint displacements and v is the member end displacements of all the member of the structure. T is a transformation matrix and only depends upon the geometry of the structure and also sometimes known as the geometric matrix.

For determinate structure the equations of equilibrium are enough for the complete stress analysis of the structure and the forces in members are independent of relative stiffnesses of the members. The compatibility equations are mere identities in this case. The member areas can be designed according to any suitable procedure and may be designed to stress the individual mebers to their maximum capacity.

For an indeterminate structure all the three categories of equations are necessary for the stress analysis of the structure. The analysis and the feasibility of a full-stress design procedure will be developed here for trusses and frames.

3.4 Feasibility of a full-stress design for trusses

A pin connected truss is a special type of structure and the load is carried by virtue of its axial stiffness. This stiffness is only a function of member areas and the length of the members. For a given goemetric configuration the variables are only the member areas and here the feasibility of a design is only with respect to areas of the members of a truss. As has already been pointed out the design of a determinate truss does not pose a problem because the areas of the members does not govern the analysis. But for an indeterminate structure the areas does influence the analysis. Hence the design of such structures are taken up here.

For the truss, the elemental deflection is given by FL/AE where A, L and E are respectively the areas, length

and elastic modulus of the material of the member. The areas may be determined as: $A = F/\sigma$, where the F is the member force and σ is the allowable stress. F and σ remain constant throughout the length of the member. Determination of area by the above procedure satisfies only the strength criterion. It is generally uneconomical if stability criterion gives a larger area of the mebers. Generally for usual columns the use of lattice and battened members give the same area as determined by strength because the introduction of such members raise the buckling strength of the column to that value due to strength as in this case one has to look for local bucklings i.e. buckling of elements in between the lacings or battens and generally Eulers formula for total buckling does not govern the design.

For a full stress of in all members the elemental deflections are given by

$$v = \frac{\sigma L}{E} \; ; \; \sigma = F/A \qquad .. \; 3.2$$

Since σ, L and E are known for the whole structure before any analysis or design, vs for all members are constants and known. According to the compatibility equations:

$$d = T^{-1} v \qquad .. \; 3.3$$

In the above equation right hand side is all known as T is the transformation matrix of the whole structure and is a constant for any given configuration. This means before the analysis of the structure for full stress or equal stress in

all members, all the joint deformations are constants and hence an analysis is not at all necessary!

This precisely means that for an indeterminate truss because of compatibility restraints it is not at all possible to achieve full stress or same stress in all the members in the structure. In a determinate structure since the compatibility equations are only identities, such a conclusion is not applicable.

For more insight to these equations the compatibility equations will be derived and then interpreted using the principle of least work or minimum strain energy or sometimes also known as the Castigliano's Second Theorem: It may be stated as follows:

In any structure the material of which is elastic and follows Hookes law and in which temperature is constant and supports unyielding, the first derivative of the Strain Energy with respect to any particular force is equal to the displacement at the point of application of that force in the direction of its line of application. Mathematically this theorem may be written as:

$$\partial U / \partial Pn = \delta n \qquad .. 3.4$$

where U is the Strain Energy of the structure and Pn any load acting in the structure and δn is the deflection at the point and along the direction of Pn.

3.5 Specialization of the Theorem for Solving Indeterminate Trusses

Let R_j designate the redundants, 'j' denotes the total number of redundants and F_i the final member forces. K_{ij} the member forces when unit forces are applied to the basic determinate structure in place of the redundant members, then

$$U = \sum 1/2E\ (F_i)^2\ L_i/A_i \qquad .. \ 3.5$$

Summation is taken over all the members including the redundant members of the structure.

$$\partial U/\partial R_j = \sum F_i L_i/A_i E \times \partial F_i/\partial R_j \qquad .. \ 3.6$$

$$F_i = P_i + \sum K_{ij} R_j \qquad .. \ 3.7$$

where P_i is the member forces due to external load in the basic determinate structure and are constants.

$$\partial F_i/\partial R_j = K_i \qquad .. \ 3.8$$

the summation drops out as, unless $i = j$, $\partial F_i/\partial R_j = 0$

K_i is the member force when only one of the R_js act.

$$\partial U/\partial R_j = \sum (P_i + \sum K_{ij} R_j)\ K_i\ L_i/A_i E \qquad .. \ 3.9$$

in the above expression the displacement along R_j direction is included i.e. for the redundant members 'j' $P_j = 0$ and $K_{ij} = K_i = 1$ and we have

$$\delta_j = R_j\ L_j/A_j E \qquad .. \ 3.10$$

which is the displacement along R_j. There are 'j' equations

of the type $\partial U/\partial R_j$ and these are the compatibility equations. The solution of $\partial U/\partial R_j = 0$ gives the Redundant forces.

$$F_i = P_i + \sum K_{ij} R_j; \quad F_i/A_i = \sigma_i(-1)^l \qquad .. \; 3.11$$

where σ_i is the stress in the members and $l = 2$ for tensile members and $l = 1$ for compression members. Substituting σ_i the fundamental equations becomes

$$\sum \sigma_i (-1)^l K_i L_i/E_i = 0; \quad \text{Let } C_i = L_i/E \text{ then}$$

$$\sum \sigma_i K_i C_i(-1)^l = 0 \qquad .. \; 3.12$$

where the summation extends over all the members of the structure.

Suppose all members are designed to achieve full stress then $\sigma_i = \sigma_p$ = A constant and can be excluded from the summation sign as $\sigma_p \neq 0$ and if E is also a constant

$$\sum_{\sigma} (-1)^l K_i L_i = 0 \qquad .. \; 3.13$$

The above relation is a geometric relation and for any arbitary configuration need not be equal to zero or it is not possible to achieve full stress in all members of the structure. Since there are only 'j' such equations only 'j' member stresses cannot be equal to the permissible values but are to be determined by the geometry. This makes the design of indeterminate trusses as complicated as its analysis.

Illustration

Suppose an indeterminate structure has m number of members and the redundancies be 'j'. For a single load

condition there are 'j' equations in m unknown stresses. Because of this one can assign any value to (m-j) member stresses and the 'j' stresses are to be obtained by solving the 'j' equations and these need not be equal to the permissible stresses. Let the 'j' stresses obtained by solving the above set of compatibility equations be b_i, in the ith member. If σ_p is the permissible value then $(f_i - b_i) \neq 0$. There are 'j' such equations to achieve full stress $f_i = b_i$ or $(-1)^1 K_i C_i = 0$. But since this is the geometry of the structure, for achieving full stress the geometry of the structure has to be predesigned. This is not the same as the 'Mitchell Structures'[36]. This deals with the configuration for a Statically Determinate Structure for a single load condition, which has the minimum volume.

All the above discussions are for one load condition and the problem of two load condition is given below.

For the same structure having m members and n redundants for two load conditions there are 2n compatibility equations although the number of redundant stresses are only n. In this case for the first load condition solve for the n member stresses such that (m-n) member stresses are equal to permissible value of the stress and the n member stresses are less than the permissible value. For the second load condition choose (m-2n) members and the 'n' member the stresses of which are not assumed at the permissible value under first

load condition and solve for the remaining 'n' member stresses such that these member stresses remain within the safe limits. There are two difficulties encountered in the above procedure.

1) The load conditions must be necessarily be two or more. For the same structure having m members and n redundants the design forces (higher of the forces under different load conditions) should be such that, at best, (m-n) members can have permissible stress under one load condition. Let

P_{1i}; $i = 1, m$ be the member forces under the first load condition

P_{2i}; $i = 1, m$ be the member forces under second load condition

P_{di}; $i = 1, m$ be the design forces in the members

For all members fully-stressed at best (m-n) P_{di}s can belong to either P_{1i} or P_{2i} separately. If say (m-n)+1 P_{di}s are from one of the load conditions then only n-1 compatibility equations are satisfied and one equation is not satisfied. Hence such a design will necessarily force one member area equal to zero reducing the indeterminancy of the structure to n-1. In short this means that in general even for multiple load conditions may not give a full stress design without forcing one or more areas to zero. For the multiple load conditions in which it is possible to design without violating the compatibility restraints will be referred to as the 'Real Multiple Load Condition' for future reference.

2) Even though apparently all the members of the structure are fully stressed under multiple load condition, a little thought will reveal that we are more or less

designing a statically determinate structure for each load conditions separately. That is to say for each load condition a part of the structure carry the load as a determinate structure and little stresses are thrown to other members to satisfy the compatibility constraints. A different approach is shown for the design of such structure in the design Chapter.

3.6 Feasibility of a Full-stress Design for Frames

We start from the equation:

$$v = Td$$

In a plane frame the aim is to achieve full stress only at one section throughout its span. The member end deformations 'v' depends upon the stress distribution throughout its span which are not equal to the permissible value. Hence forcing one section to its full stress limit the displacements v are not determined and the compatibility equations are not violated.

As was done before for trusses the compatibility equations from Castiglianos Second theorem may dbe derived as

$$(M_i + \sum_{j=1}^{N} m_i R_j) \; m_i \; ds/E_iI_i + (P_i + \sum_{j=1}^{N} K_i R_j) \; K_i \; ds/A_iE_i$$

$$+ \; K \; (V_i + \sum_{j=1}^{N} S_i R_j) \; S_i \; ds/A_iG_i = 0 \qquad .. \; 3.14$$

The integration extends over all the members of the entire structure. M_i, P_i, V_i are respectively the bending moment;

axial sforce and shear forces for the ith member when all redundants are removed. m_i and S_i have the same meaning as k_i and represent for unit moment and unit shear forces. ds is the length element A, E, G, I are the properties of the members and have the usual meaning and are defined earlier.

The compatibility equations represent the deformation of a structure. For flexural members the deformations are predominantly due to flexure and direct stress or shear contributions to the deformations are negligible. For simplicity only flexural deformation may be taken into account. This assumption reduces the above equations 3.14 to

$$(M_i + \sum_{j=1}^{N} m_i R_j) \; m_i \, ds/E_i \, I_i = 0$$

Let $F_i = M_i + \sum_{j=1}^{N} m_i R_j$ where F_i is the final moment at any point in a structure. Then theabove equation reduces to

$$F_i \, m_i \, ds/E_i \, I_i = 0; \quad F_i/I_i = \sigma_i/Y_i$$

$$\text{or} \quad \sigma_i/Y_i \, m_i \, ds = 0 \qquad .. \; 3.15$$

In the above equation σ_i is the stress at any point of a member. Our aim is only to fully stress only one section i.e. for one section of each member $\sigma_i = \sigma_p$. This restriction in no way effects the integral being not equal to zero. The stresses at all other points of a member can so adjustedasto make the integral equal to zero.

Even for one load conditions the compatibility constraints are not at all violated. This means for rigid jointed plane frames and also for space frames it is possible to full-stress one section of each member even under one load conditions. Although the analysis of such structures are complicated the design is not very different from that of a determinate structure except that there are member end forces.

CHAPTER IV

DESIGN OF STRUCTURES AND STRUCTURAL COMPONENTS

4.1 Approximate Analysis for Rigid Jointed Plane Frames

For the exact analysis both the areas and moment of inertias of all the elements of the structure are necessary. These parameters influence the convergency of the iterations. A better guess of these parameters may save considerable labour and computer time. Hence by some approximate method these parameters are evaluated.

A plane frame has to resist both lateral and gravity loads. Since it is a linear structure the influence of lateral loads and gravity loads are determined by approximate methods separately and the solution superimposed to determine the total forces in the members.

4.2 Analysis for Lateral Loads

A tall structure is generally subjected to high wind loads. To simplify the analysis it will be assumed that the wind loads are concentrated at the floor levels.

For this type of loads the structure may be assumed to behave as a Cantilever and the stresses are evaluated by ordinary beam theory. The method will be developed first and the assumptions will be made as and when necessary and finally they will be listed at the end of this section.

For a Cantilevered beam the bending moment M and Shear force V for any section can be found out by statics.

The stresses are evaluated by beam theory as:

$$\sigma = MY/I; \quad S = V/A \qquad .. \ 4.1$$

where σ is the normal stress at a distance Y from the neutral axis, S is the average shear stress assumed constant throughout the section, A and I are respectively the area and the moment of inertia of the cross-section.

The Cantilever analysis of a building frame is exactly similar to the Cantilevered beam. The moment 'M' and the shear force V are the storey moment and the storey shear. Storey moment is the moment due to all loads acting above the storey level and storey shear is the algebraic sum of all loads above the storey level. Just as the moment varies from point to point along the Cantilevered beam so does the storey moments vary within a storey being maximum at storey bottom and minimum at the floor level of the storey. To get rid of this difficulty storey moments and storey shear forces are evaluated at the middle of the storey and assumed constant for whole of the storey. The normal stress and the shear stress of the Cantilevered beam becomes the axial stress and the shear stress of the columns of the storey under consideration. Since the shear stress distribution is assumed constant across the cross-section of the Cantilevered beam the shear stress along the column section remain the same. If it is further assumed that the columns for a storey has some areas then the storey shear is equally divided among all the columns of the storey.

$$S_r = V_r/N_r \qquad .. \ 4.2$$

where S_r and N_r are respectively the shear force of a column at rth storey and number of columns of the storey. S_r the column shear force remain constant for all the columns of the storey.

To determine the column axial force the moment of inertia of the column group of rth storey about their centroidal axis has to be evaluated. The centroidal axis passes through the centre of gravity of all the columns of the storey under consideration. If the columns have same areas as is assumed, then the centroidal axis passes through the mid-point of the span of the storey under consideration if columns are symmetrically located about this point.

$$I_r = \sum A_i \ R_i^2 = A \sum R_i^2 \qquad .. \ 4.3$$

where I_r is the moment of inertia of the rth storey and A_i is the area of the columns of the same storey and assumed constant and equal to A. R_i is the distance of ith column of rth storey from the centre of gravity of the storey.

$$P_i = \sigma_i \ A = A \ MR_i/A \sum R_i = MR_i/\sum R_i \qquad .. \ 4.4$$

where P_i is the column axial force for ith column of rth storey.

Since R_i is a constant for a storey the axial force varies as the distance from the centroidal axis. This is analogous to normal stress in a beam which varies linearly from the neutral axis.

The wind load is assumed to act only at panel points and the span of the members are free from load. If the beam girders are assumed to be rigid compared to the columns, then the columns carries moments due to sway only. For such a case the column moments and beam moments change sign once along their span. Since the aim here is to determine the approximate values of the stress-resultants it may logically be assumed that the contraflexure points for wind load alone may lie at the centre of the beam span or column height. Once the hinge points are assumed the stress analysis can be done by statics alone.

$$CM = S \times H/2; \; BM = CM_r + CM_{r-1}; \; BS = \; (P_{ir} - P_{ir-1}) \qquad .. \; 4.5$$

where CM = The end moment of a column

S = Column shear evaluated earlier

H = Height of column

BM = End moment of a beam

CM_r and CM_{r-1} are respectively the column end moments attached to the beam from r and r-1 storey.

BS = Shear force of a beam

The summation sign   is over all the columns of rth storey.

P_{ir} and P_{ir-1} are the column axial force for ith column of r and r-1 storey to which the beam belongs.

Thus the stress analysis for the entire structure from wind loads is evaluated.

4.3 Assumptions in the Cantilever Analysis

The main assumptions may be listed thus:

(a) Wind loads are concentrated at the floor levels only.

(b) All columns of a storey carry the storey shear according to their areas. If areas of all columns are assumed equal the shear is equally divided among all the columns of the storey.

(c) The axial force of a column varies directly as the distance from the centre of gravity of the column group of the storey.

(d) There lies hinge points at the mid-height of columns and mid-span of each beam.

4.4 Limitations

This method may not be applied to a building frame where the beams are not continuous over all the columns of a storey i.e. column belongs to two stories. The method cannot be applied to building frames in which the column heights vary with in a storey as the storey moment and storey shear cannot be evaluated as before. But the method can be applied to any frame where the beams runs over all the columns having same height in a storey. There is no restriction for the number of columns in a storey.

4.5 Gravity Load Analysis

Since the beams are made up of mild steel and the section remains constant throughout the span, the sign of the bending moments are not of importance to the designer except

for the possibility of some local instabilities. The magnitude of the design bending moment may lie somewhere in between simply supported bending moment and fixed end bending moment. For initial design the beams are designed for 120% of fixed end moments. It is further assumed that the columns carry only the beam reactions and there is no distribution of fixed end moments in between beams and columns.

The design load for any member is the addition of the absolute values of wind forces and gravity forces for the member.

4.6 Determination of Approximate Values of the Displacements

Although a free joint of a regular building frame has three degrees of independent deformations, the axial deformations are negligible when compared to the sway and rotations except for the flower stories of a multi-storied building. The convergency of these displacements is not at all a problem and iterations may be stopped if the rotations and sways have stationary values. In this approximate analysis if axial defomations are assumed zero then the slope deflection equations for a member I-KI becomes;

$$\begin{bmatrix} P(I,1) \\ P(I,2) \end{bmatrix} = \begin{bmatrix} 4EI/L & 6EI/L^2 \\ 6EI/L^2 & 12EI/L^3 \end{bmatrix} \begin{bmatrix} U(I,1) \\ U(I,2) \end{bmatrix} + \begin{bmatrix} 2EI/L & -6EI/L^2 \\ 6EI/L^2 & -12EI/L^3 \end{bmatrix} \begin{bmatrix} U(KI,1) \\ U(KI,2) \end{bmatrix} \quad \text{.. 4.6}$$

It is proved in the numerical analysis textbooks that convergency of iterations of a set of equations will

definitely occur if the absolute values of the coefficients on the main diagonal is greater than the sum of the absolute values of the other coefficients in a given row.

It may be clearly noted that all structural analysis problems need not satisfy the above theorem. Hence although the problems will converge for stable structures the convergency may be extremely slow. To illustrate this two problems are considered below.

Problem 1. (Page 180 and 176 - No. 64 and 63 respectively from Statically Indeterminate Structure by C.K. Wang)

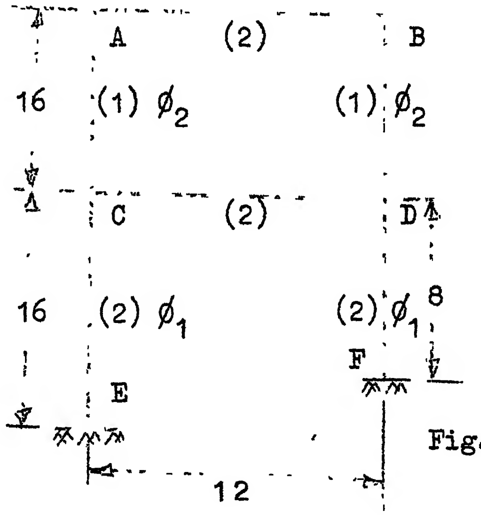
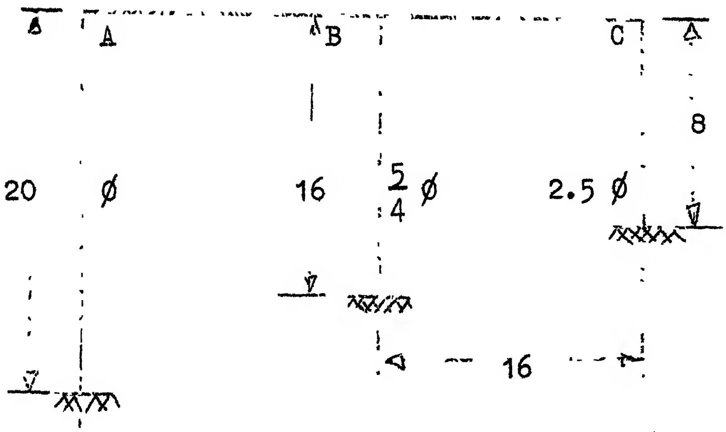


Fig. 4.3



Problem 1

Since axial deformations are neglected the number of independent displacements are:

$$(\theta_A \;\; \theta_B \;\; \theta_C \;\; \theta_D \;\; \Delta_1 \;\; \Delta_2)$$

as shown in the diagram. The total stiffness matrix for the above order of displacement is:

$$A = \begin{bmatrix} 6 & 2 & 1 & 0 & -1/16(-1) & 0 \\ 2 & 6 & 0 & 1 & -1/16(-1) & 0 \\ 1 & 0 & 10 & 2 & -1/16(-1) & 2 \\ 0 & 1 & 2 & 10 & -1/16(-1) & 4 \\ -1 & -1 & -1 & -1 & 1/12(3/4) & 0 \\ 0 & 0 & -2 & -4 & 0 & 5/12 \end{bmatrix} \begin{bmatrix} \theta_A \\ \theta_B \\ \theta_C \\ \theta_D \\ \Delta_1 \; (\phi_1) \\ \Delta_2 \; (\phi_2) \end{bmatrix}$$

$$\text{where } \phi_1 = \Delta_1/H = \Delta_1/16 \qquad \phi_2 = \Delta_2/16 \qquad \text{.. 4.7}$$

Problem 2

The deformation vector is:

$$(\theta_A \;\; \theta_B \;\; \theta_C \;\; \Delta)$$

as shown in the diagram.

$$B = \begin{bmatrix} 46 & 15 & 0 & -2/5\ (-8) \\ 15 & 80 & 15 & -5/8\ (-12.5) \\ 0 & 15 & 70 & -2.5\ (-50) \\ -32 & -50 & -200 & 19.9\ (398) \end{bmatrix} \begin{bmatrix} \theta_A \\ \theta_B \\ \theta_C \\ \Delta\ (\phi) \end{bmatrix} \qquad \text{.. 4.8}$$

$$\phi = \Delta/H = \Delta/20$$

A glance of the matrix A reveals that it has two extremely poor diagonal element namely a_{55} a_{66}. The last two equations of 'A' matrix in the classical analysis is known as the storey shear equations and these involve the sways deformations.

The condition is not so severe for the stiffness matrix 'B' of Problem 2. Here the diagonal elements are bigger than the off diagonal terms except for the shear equation (last equation in the 'B' matrix). Although the convergence may be better in this case but still may be poor.

Convergence for sway deformations becomes extremely slow for single bay multi-storeyed buildings because of the shear equations. The convergency improves if the number of bays increases but still may be poor.

4.7 Improvement in the Convergence of Deformations

The convergence may be improved as follows:

(a) Instead of $\triangle$ if ϕ is assumed as the unknown as shown in the figure the coefficient in the 'B' matrix of problem 2 becomes bigger and the diagonal terms form a real strong diagonal improving convergency. But for problem 1 the change in convergence may not be significant.

(b) For a regular multi-storeyed building the sways or shear equations may be directly inverted and for rotations iteration may be resorted to. This may not load the computer memory as it may be assumed that each storey has only one sway.

(c) If a good guess may be done for the initial sways then also the convergence may be accelerated. These are taken from papers published in the early 20th century. Two of the methods are illustrated below.

(i) Here the assumptions are:

(1) Major portion of the sway is only due to wind load.

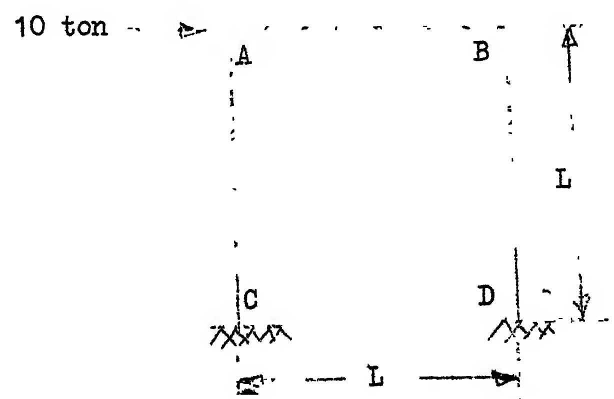
(2) The floors are rigid.

For such a case sway for each storey may be calculated as:

$$M = 6\ EI\delta/L^2 \text{ or } \delta = ML^2/6EI \qquad .. \ 4.9$$

Since moment for the columns are determined, and they are designed, can be found for any column of a storey and may be assumed constant for all the columns of the storey. The sway for the nth storey will be the sum of all the storey sways up to that storey. This may give exact results and are very useful for single bay multi-storey frames. Since the beams are assumed rigid the actual sways will become larger than the above calculated sways so it is better to assume a 20% increase for all the sways.

Illustration



Analysis by cantilever method gives Bending Moment, 300 ton inch in columns.

$$= ML^2/6EI = 0.5'' \text{ if } E = 13{,}000$$

at 20% increased sway becomes 0.6".

The exact sway deformation even taking axial deformation = 0.64".

The stiffness matrix for the above problem for θ and becomes:

$$K = \begin{bmatrix} 10\ I/L & -6\ I/L^2 \\ -6\ I/L^2 & 12\ I/L^3 \end{bmatrix} \text{ or } \begin{bmatrix} 8.7 & -0.036 \\ -0.036 & 0.0005 \end{bmatrix}$$

where I/L = 125/144 = 0.87.

As can be seen it ihas very weak diagonal elements and the sway displacement does not even converge after 50 cycles of iterations, but when initial sway as calculated above is introduced the convergence is achieved within four cycles. This method is adopted for the frame analysis because of its simplicity.

(ii) Here the assumptions are:

(a) Axial deformations are neglected.

(b) Sway for a storey remains constant.

(c) Joint rotations for a storey remains constant for all the joints of the storey.

(d) To start with rotation for n-1 and n+1 storey is assumed same as the rotation of nth storey.

Procedure

(1) Determine the external storey moment as was done in the Cantilever method. Let this moment be M_l and M_r for two consecutive storeys.

(2) Determine the internal moments of a storey. Moment for one column $= M = M_{AB} + M_{BA} = K(6\theta - 6R)$

where θ is rotation of the storey and $R = \Delta/H$ — sway angle of the storey. Δ is the sway and H is the height of the storey.

Total moment of resistance of one complete storey $= M_s = \sum M = \sum K\ (6\theta - 6R)$. The summation extends over all the columns of the storey. Similarly

$$M_l = \sum K_l\ (6\theta_l - 6R_l) \text{ .. for lth storey}$$

$$M_r = \sum K_r\ (6\theta_r - 6R_r) \text{ .. for rth storey}$$

Adding and equating $\theta_r = \theta_l = \theta$

$$M_l + M_r = 6(\sum K_l + \sum K_r)\ \theta - 6(\sum K_l\ R_l + \sum K_r\ R_r) \qquad .. \ 4.10$$

(3) Above is the storey equation. Now joint equilibrium equations become

$M = 0$; summation extends over all members meeting at the joint.

Equation for joint 'A'

$$K_{AC}\ 2\theta_A + K_{AB}\ 2\theta_A + K_{AM}\ 2\theta_A - 3R_{AM}\ K_{AM} - 3RA_B\ K_{AB} + K_{MA}\ \theta_M + K_{BA}\ \theta_B + K_{CA}\ \theta_C = 0$$

This may be rewritten as:

$$2\theta_A\ (K_{AC} + K_{AB} + K_{AM}) - 3(R_{AM}\ K_{AM} + R_{AB}\ K_{AB}) +$$

$$(K_{MA}\ \theta_M + K_{BA}\ \theta_B + K_{CA}\ \theta_C) \quad = \quad 0$$

For the whole storey

$$2\theta_A(\sum K_{Beam} + \sum K_{Col}) -3(\sum K_1\ R_1 + \sum K_r\ R_r) +$$

$$(K_{MA}\ \theta_M + K_{BA}\ \theta_B + K_{CA}\ \theta_C) \qquad .. \ 4.11$$

where $\sum K_{Beam}$ = twice the total stiffness of beams of a floor.

$\sum K_{Col}$ = total stiffness of column of a storey

K_1 = total stiffness of all columns of upper storey

K_r = same for lower storey

if $\theta_M = \theta_N = \theta_o = \theta_p = \theta_q = \theta_r = \theta_1$ and

$\theta_B = \theta_D = \theta_F = \theta_H = \theta_J = \theta_L = \theta_r$

$$(\sum K_{MA}\ \ \theta_M + K_{BA}\ \theta_B + K_{CA}\ \theta_c) = \theta_1 \sum K_1 + \theta_r \sum K_r + \theta_A/2 \sum K_{Beam}$$

Substituting the above simplifications in the joint equilibrium equations for all joints of the storey becomes

$$2\theta_A\ (1.5 \sum K_{Beam} + \sum K_{Col}) + \theta_1 \sum K_1 + \theta_r \sum K_r -3(\sum K_1\ R_1 + \sum K_r\ R_r) \qquad .. \ 4.12$$

Now there are two equations for each storey namely

1. $\sum K_1\ R_1 + \sum K_r\ R_r = (M_1 + M_r)/6 - \sum(K_1 + K_r)\ \theta_r$

2. $\sum K_1\ R_1 + \sum K_r\ R_r = 2/3\ (1.5 \sum K_{Beam} + \sum K_{Col})\ \theta_r +$

$$1/3(\sum \theta_1\ K_1 + \sum \theta_r\ K_r)$$

This may be rewritten as:

$$2\theta_A\ (K_{AC} + K_{AB} + K_{AM}) - 3(R_{AM}\ K_{AM} + R_{AB}\ K_{AB}) +$$

$$(K_{MA}\ \theta_M + K_{BA}\ \theta_B + K_{CA}\ \theta_C) \quad = \quad 0$$

For the whole storey

$$2\theta_A(\sum K_{Beam} + \sum K_{Col}) - 3(\sum K_1\ R_1 + \sum K_r\ R_r) +$$

$$(K_{MA}\ \theta_M + K_{BA}\ \theta_B + K_{CA}\ \theta_C) \qquad .. \ 4.11$$

where $\sum K_{Beam}$ = twice the total stiffness of beams of a floor.

$\sum K_{Col}$ = total stiffness of column of a storey

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if $\theta_M = \theta_N = \theta_o = \theta_p = \theta_q = \theta_r = \theta_1$ and

$\theta_B = \theta_D = \theta_F = \theta_H = \theta_J = \theta_L = \theta_r$

$$(\sum K_{MA}\ \ \theta_M + K_{BA}\ \theta_B + K_{CA}\ \theta_c) = \theta_1 \sum K_1 + \theta_r \sum K_r + \theta_A/2 \sum K_{Beam}$$

Substituting the above simplifications in the joint equilibrium equations for all joints of the storey becomes

$$2\theta_A\ (1.5\sum K_{Beam} + \sum K_{Col}) + \theta_1 \sum K_1 + \theta_r \sum K_r - 3(\sum K_1\ R_1 + \sum K_r\ R_r) \qquad .. \ 4.12$$

Now there are two equations for each storey namely

1. $\sum K_1\ R_1 + \sum K_r\ R_r = (M_1 + M_r)/6 - \sum(K_1 + K_r)\ \theta_r$

2. $\sum K_1\ R_1 + \sum K_r\ R_r = 2/3\ (1.5 \sum K_{Beam} + \sum K_{Col})\ \theta_r +$
$1/3(\sum \theta_1\ K_1 + \sum \theta_r\ K_r)$

Eliminating $\sum K_l R_l + \sum K_r R_r$ from these two equations

$$(M_l + M_r)/6 - (\sum K_l + K_r)\,\theta_r = 2/3\,(1.5\sum K_{Beam} + \sum K_{Col})\theta_A + $$
$$+ 1/3\,(\sum K_l\,\theta_l + \sum K_r\,\theta_r) \qquad ..\ 3.13$$

If for the initial interation all θ_s are assumed same and equal to θ_o then

$$\theta_o = (M_l + M_r)/6\sum K_{Beam} \qquad ..\ 4.14$$

Let $\sum K_{Girder} = 1/2\sum K_{Beam}$ then

$$\theta_o = (M_l + M_r)/12\sum K_{Girder} \qquad ..\ 4.15$$

Once θ_o is known R_l can be found out from the storey moment equations.

Few iterations for each storey may given a fairly good solution.

This method is due to John E. Goldberg (45).

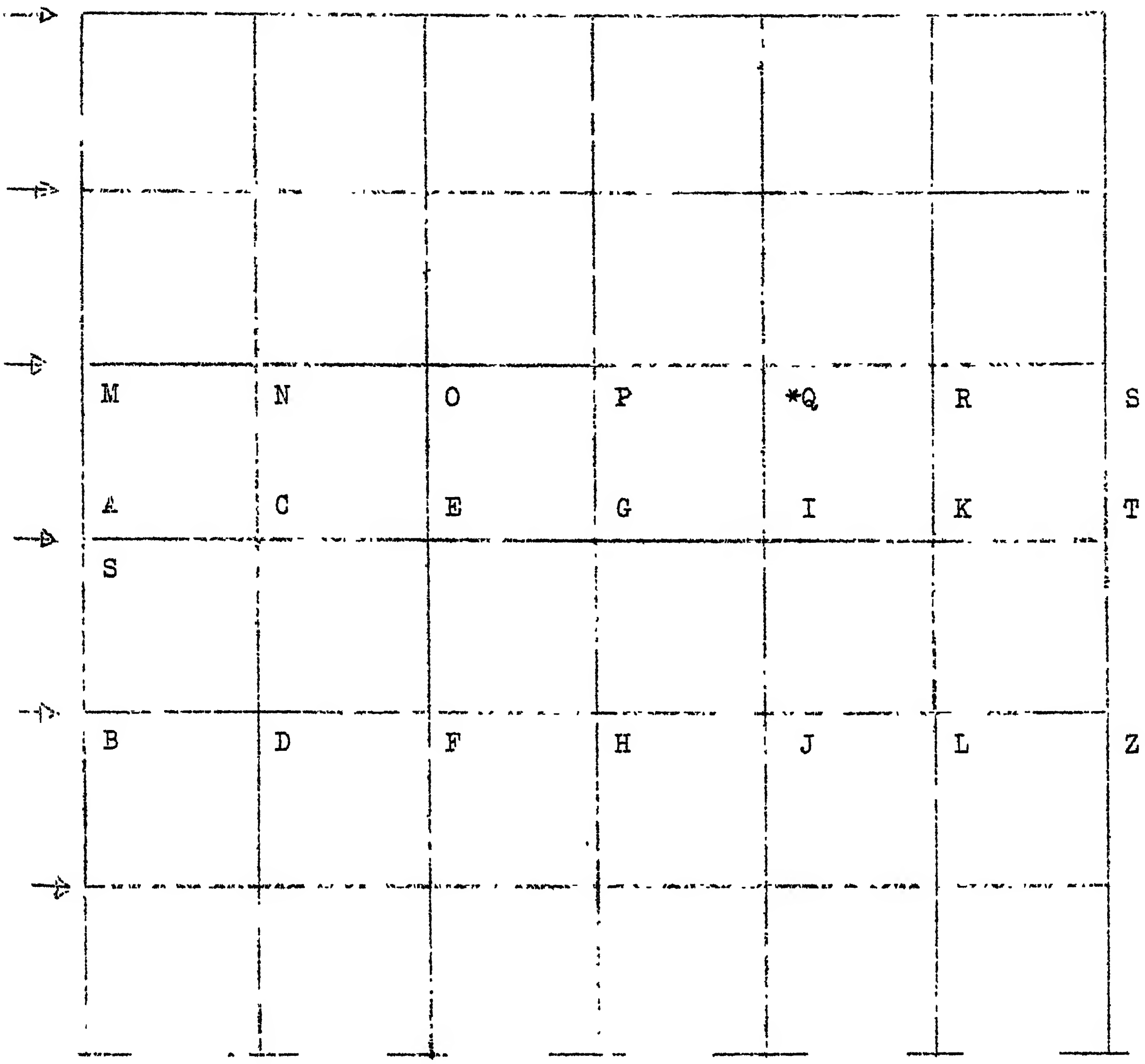
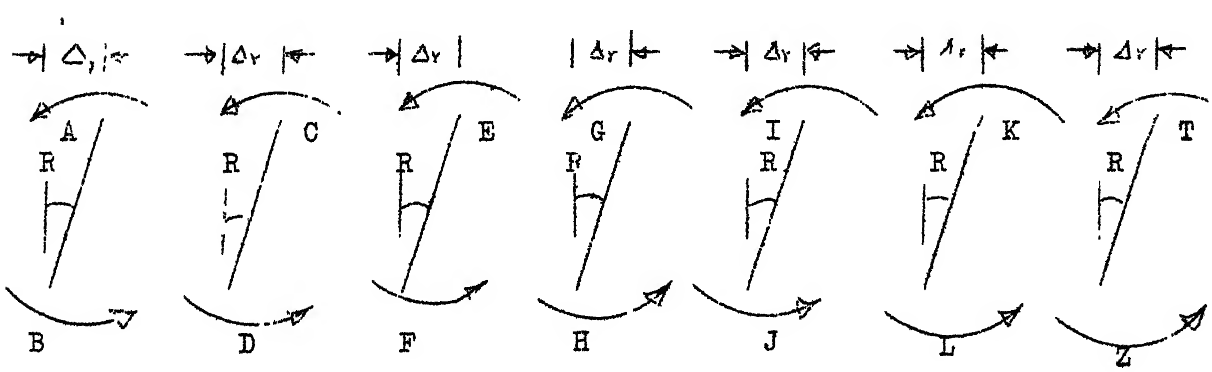


Fig. 4·6



INTERNAL MOMENTS OF RESISTANCE OF A STOREY

4.8 Design of Plane Frame Members

The object of a design is to provide members in a structure to satisfy the following requirements:

(1) Serviceability or functional requiremnets

(2) Economic requirements

(3) Strength and stability requirements

4.9 Serviceability or Functional Requirements

The elements of a structure must safely discharge the function for which it may be built i.e. the serviceability criterion is satisfied. For example the water tank has to retain water i.e. the cracks in the tension zone of the concrete in the tank should not be of such magnitude which would allow leakage. Similarly the deformations of a beam supporting a glass panel must be small enough to safely support over head panel.

The serviceability of a plane frame generally refers to the deformation which should be within a permissible value satisfying the functional requirements. The permissible deflection of a beam is usually relative to the span but sometimes it may be independent of the span if it has to discharge some special requirements. For the above example of the glass panel over the beam the deflection has to be such that no damage is caused to the structure above and hence it has to be independent of the span. But these are special cases and the design here is governed by the general case.

The Indian Standards Specification (IS-800) limits the maximum deflection to span/325.

The correct determination of the deflection is as difficult as the analysis itself. Instead deflection will be related to the depth of a simply supported beam with an uniformly distributed load throughout its span. Since the deflection of the members a rigid frame is expected to be less than the simply supported beam this calculation keeps the design on the safe side.

$$\delta_{max} = (5\ ML^2)/(48\ EI); \quad \sigma_{max} = Md/2I$$

$$\delta_{max} = (5\sigma_{max}L^2)/24\ Ed$$

if $\delta_{mas}/L = 1/325; \quad = 20$ ksi; $E = 30{,}000$ ksi

$$L/d = 22.2 \qquad ..4.13$$

where $M = wL^2/8$; w being the uniformly distributed load over the span L; δ_{max} is the central deflection; and d is the depth of the beam; E,I has the usual meaning.

If the depth of the beam is at beast 1/22.2 of its span the criterion of serviceability is satisfied for a mild steel beam. Since the economic depth of the beam generally lies in between 1/15th to 1/10th of its span serviceability is not at all a problem for mild steel beams.

4.10 Economic Requirements

According to this requirement, for a given bending moment, shear force, axial force and span, the section designed

is economical when the area of the section is a minimum. Instead of volume, area is taken as the criterion because the section is kept constant throughout the span of the member. An I-section is designed for the elements of a plane frame. For such an element initially the area of the section reduces as the depth of the section increases from serviceability depth to a certain depth and thereafter because of local and total instabilities of the member, the area starts increasing for any increase of depth. The depth at which the area is a minimum is termed as the economical depth. The aim here is to achieve this area.

For a particular depth higher than the serviceability depth the area of the I-section is determined which satisfies the strength and stability criteria. The above procedure is repeated for several depths until the economic depth is attained. This gives the minimum area of the member.

4.11 Strength and Stability Requirements

This requirement is, in fact, the prime criterion for the design of a structural element of a building frame. In building frames, the structural members are generally classified as beams and columns according to their position and type of the predominant loads. In the general frame design such a classification is not necessary, instead a general formulation is made to design an uniform I-section throughout the span to resist the given forces and automatically either a beam or a column is designed according to the forces the section has to carry.

For a particular depth, d, a uniform I-section is designed to satisfy the strength and stability criteria.

The problem here is to design an I-section for a given set of loads namely the bending moment, the axial force and the shear force acting on a span of length L. Each elements of the I-section namely the Web and the Flanges are designed for strength and stability.

4.12 Strength Criteria

For an element in direct compression and bending about one axis the criteria of strength is governed by the interaction formula:

$$f_a/F_a + f_b/F_b \leq 1 \quad \text{.. 4.14}$$

where f_a and f_b are respectively the axial and bending stresses of the section and F_a, F_b are respectively the permissible values for the same stresses. The interaction formula is satisfied by iterating on the area of the section.

For the initial determination of f_a and f_b it is assumed that the flanges carry the total bending moment and 2/3rd of the axial forces, f_a and f_b are, proportional to the bending and axial forces in the flange; and

$$F_a = F_b = \sigma ,$$

being the permissible value based on yield criterion.

Since $F_a = F_b$ the iteraction formula becomes

$$f_a + f_b \leq \sigma$$

$$\text{Force in each flange due to moment} = M/d$$

$$\text{Force in each flange due to axial force} = P/3$$

$$(M/d)/(P/3) = K = f_b/f_a; \quad f_b = 3M\, f_a/P\, d$$

$$f_a + f_b = f_a\,(1 + 3M/P\, d) = \sigma$$

$$f_a = (\sigma\, P\, d)/(P\, d + 3M) \qquad .. \; 4.15$$

$$f_b = (3M\, \sigma)/(P\, d + 3M) \qquad .. \; 4.16$$

The above formulae are necessary for a preliminary value of the section area. The initial area is higher of the two values

$$A = P/f_a \quad \text{or} \quad A = (1.2\, M/d + F)/$$

where F is the direct compressive force.

Since the section designed is an I-section the approximate moment of inertia is given by

$$I = 0.38\, A \times d^2; \qquad Z = 2I/d$$

$$f_a = P/A; \qquad f_b = M/Z$$

Let $\quad BETA = f_b/F_b + f_a/F_a$

The value of BETA determines the safety of the section against strength. The structure is safe or unsafe according to whether BETA is higher or lower than one. In both cases if area is increased BETA times the iteraction formula is identically equals to unity and the section is safe.

4.13 Design of the Web Plate

The section designed is an uniform I-section without any vertical or horizontal stiffness. The web

of the I-section has to be checked for buckling due to shear, bending and axial compression.

The critical stress at which buckling is imminent for a rectangular plate with various boundary conditions and various load conditions for load in its own plane is given by the formula

$$S_{cr} = (K \pi^2 E)/(12(1-\nu^2)(b/t)^2) \; * \qquad .. \; 4.17$$

where K = a constant dependent upon boundary conditions and load conditions (shear, bending or axial load). E and are Youngs modulus and Pinssons ratio; t = thickness of the plate; b = length of the loaded edge of the plate, except when the plate is subjected to pure shear it is the smallest lateral dimension.

Substituting for E = 30,000; ν = 0.3

$$S_{cr} = K \; 27100/(b/t)^2 \qquad .. \; 4.18$$

4.14 Determination of the Thickness of the Web Plate To Prevent Shear Buckling

In this case the boundary is taken to be simply supported along all edges and a/b or span/depth >5. For the above conditions the value of K is a minimum.

---

*This formula along with other formulae of this Chapter are quoted from the design of steel structures by Gaylord and Gaylord.

For this case the K value for pure shear load is K = 5.3 and upon substitution:

$$v_{cr} = 144000/(d/t)^2 \quad .. \; 4.19$$

Allowing a factory safety of 1.66 against buckling

$$v_{al} = 87000/(d/t)^2$$

Hence the allowable stress is either 'v_{al}' or permissible value of shear stress based on yield criterion for mild steel. For safe design the actual shear stress should at best be equal to the allowable shear stress.

$$v_{max} = 1.25\ V/dt_1$$

$$t_1 = (1.25 \times V \times d/87000)^{1/3} \quad .. \; 4.20$$

The factor 1.25 is used because the shear stress V/dt_1 is the average value, whereas the maximum shear stress for an I-section is generally 1.25 times the average shear stress. If the web has a thickness t_1 it can carry safely the shear force V without buckling in shear.

4.15 Buckling due to Bending and Axial Force

The section is subjected to normal stress due to both bending moment and axial compression. This normal stress due to both the effects must lie within the permissible limits.

The web plate of the I-section can buckle either in bending or in direct compression. The thicknesses for the

web plate are determined for both the cases separately so as to achieve permissible stress based on yield criterion without buckling. Since the permissible value for mild steel is same both in bending and axial stress, the design thickness is taken as the sum of the two thicknesses.

i) Bending Buckling

Substituting suitable value for K the allowable stress in bending for a web plate to prevent buckling is given by

$$s_{cr} = 650{,}000/(h/t)^2 \qquad \text{.. 4.21}$$

If a factor of safety of 1.33 is taken against buckling due to bending and maximum allowable stress is taken as 20 ksi

$$(h/t) = 173 \quad \text{or } t_b = h/173 \qquad \text{.. 4.22}$$

Hence if the depth is within 173 times the thickness of the web bending buckling does not occur and maximum allowable stress of 20 ksi may be achieved with a factor of safety of 1.33. A low factor of safety of 1.33 is assumed because the 'K' value is taken for a plate with all sides free, and this is a conservative value for the web of an I-section.

ii) Compression Buckling

Substituting proper value of K, the allowable stress i.e. compression to prevent buckling due to compression becomes:

$$s_{cr} = 204000/(h/t)^2 \qquad \text{.. 4.23}$$

To achieve a 20 ksi stress in compression with a factor of safety of 1.33

$$h/t = 87 \text{ or } t_c = h/87 \quad .. \; 4.24$$

t_b and t_c are the two bounds for the web thicknesses for pure compression or pure bending. In the actual case since both bending and axial forces act, the thickness for both the cases may be determined substituting the actual stress in the web due to bending and due to compression.

$$h/t_c = 390/(f_c)^{1/2} \text{ or } t_c = h(f_c)^{1/2}/390$$

$$\text{and} \quad t_b = h(f_b)^{1/2}/775 \quad .. \; 4.25$$

where f_c and f_b are respectively the actual stresses in the web which are calculated previously.

The thickness to prevent buckling due to bending or due to axial compression is avoided by taking the sum total of the two thicknesses as the allowable value for mild steel section due to direct compression or bending is the same.

The above formulae are derived to prevent buckling due to normal stresses or shear stresses.

In a rigid jointed frame the members are generally subjected to high moments and concentrated shear forces at the ends. These give rise to combined shear and direct stresses. For such cases it is extremely difficult to specify the exact thickness of the web to avoid buckling due to the combined

action of the shear stress and normal stresses. Since the thickness to prevent shear buckling is usually small and is generally less than half the minimum thickness required by the I.S. Code of Specifications (which specifies 0.25" as the minimum) t_1 will be taken as $2t_1$. In this case the web shear stress will become

$$v/v_{crt} = 1/0.25 \quad \text{or} \quad v = v_{crt} \times 0.25$$

The interaction formula when both normal and shear stresses act is given by

$$(v/v_{crt})^2 + (s/s_{cr})^2 \leq 1 \qquad ..44.26$$

where s and v are the bending and shear stress of the section, s_{crt}, v_{crt}, the allowable values for the same. Substituting $v/v_{crt} = 0.25$; $s/s_{crt} = 0.95$.

By providing twice the thickness for shear requirements, the normal stress only has to be lowered by 5% and for all practical purpose if the thickness due to normal stress is determined assuming the shear force zero the section designed is safe.

4.16 Design of Flanges

The flanges carry a major part of the bending moment and axial force. The area of the flange may approximately be determined as

$$A_f = (1/\sigma)\ (M/d + F/3) - 1/3\ A_w \qquad ..\ 4.27$$

In the above expression the bracketed term refers to the approximate force the flanges may have to carry if the web does not carry any bending moment 'M' and 1/3 Axial force 'F' is being carried by the flanges. But since web carry a part of the bending mcment,one third the area of the web is deducted as the contribution of the web for carrying the bending moment. In the textbooks the web contribution to resist bending moment is generally taken as one sixth the web area but for shallow depths and for low bending moments the web contribution may be as high as 1/3 itsarea.

4.17 Buckling of Flanges

Both direct load and bending moment causes direct stresses in the flanges of an I-section. The compression flange may buckle due to the combined compression. The critical compressive strength of a beam flange is given by

$$\sigma_c = (\pi d)(E\, I_y\, GJ)^{1/2})/(2I_x\, L) \qquad \text{.. 4.28}$$

where $I_x = 2\,bt^3/3$; $I_y = tb^3/6$; $J = 2bt^3/3$

Substituting for $E = 30{,}000$ ksi and $G = 12000$ ksi for mild steel and with a factor of safety of 1.66.

$$\sigma_{al} = 12{,}000/(Ld/bt) \qquad \text{.. 4.29}$$

where L, d are the span and depth of the beam; b, t are the breadth and thickness of the compression flange.

In determing the area of the flange by equation 4.27 proper value for is substituted. This value is decided

on the basis of yield or on the basis of buckling due to compressive stresses. The lower value of σ is used. This keeps the section in the safe side.

Once the flange and web elements are designed the exact moment of inertia I, the area A and the radius of gyration r are calculated.

4.18 Compression Buckling of the Section as a Whole

The critical stress in a column is generally expressed by the Euler's formula as

$$\sigma_{crt} = \pi^2 E/(L_e/r)^2 \qquad .. \ 4.30$$

The parameter L_e is substituted to take care of the proper boundaries. For non-sway cases the value of L_e is generally smaller than the clear span of the column. But for sway case it depends upon the factor

$$G = (I_c/L_c)/(I_g/L_g) \quad (46) \qquad .. \ 4.31$$

where the subscript c refers to the column and g to the beams. In a frame the girders are heavy and since a part of the slab over the girder usually act in combination the stiffness value of it is extremely high. Due to this fact the factor G becomes extremely small for usual building frames. For G equal to infinity the effective length becomes twice the clear length and for G equal to zero L_e becomes the clear span. In our case since G is small L_e is taken as the clear span. This is also as per the I.S.I. 800 Specifications.

Finally the interaction formula is satisfied i.e.

$$(P/A)/\sigma_{al} + (M/Z)/\sigma_b$$

must be equal to 1 within 5% accuracy. If theinteraction formula is not satisfied within 5% tolerance, the area is either increased or decreased in steps of few percents of the total area and the whole process repeated until the relation is satisfied.

This gives a section for a particular depth. As already mentioned under economcc requirements the process is repeated for higher depths until the minimum area of the section is achieved.

4.19 The Design of a Section may be summarized as follows:

i) Choose a depth to satisfy the serviceability requirements. Let this depth be 'd' = span/22.

ii) Determine the thickness of the web according to the equations.
4.20, 4.22, 4.24, 4.25 and satisfying I.S. Specifications for minimum conditions.

iii) Determine the area of the flange from equation 4.27. Assume a thickness (minimum 0.25) and find the spread of the flange. This spread must not be greater than 16 times the thickness (Code requirement: I.S. 800)

iv) Determine actual area and moment of inertia of the section.

v) If 'BETA' is higher than 1 the section is unsafe and increase the area of the section. Distribute this increase in area in the web or flanges according to their areas.

If BETA is less than 1 the section is oversafe. In this case the process is repeated and checked if any of the areas of the flange or the web can be reduced without violating any of the buckling or strength constraints.

vi) Repeat the whole procedure for higher depths until the area of the section has an absolute minimum value.

4.20 Design of Trusses

A preliminary design is necessary to accelerate the rate of convergence in the analysis and in the design. The convergency of the analysis depends upon the magnitude of the diagonal elements of the stiffness matrix. When all the areas of the members of a truss are assumed equal, the relative magnitudes of the diagonal elements of the stiffness matrix remains unaltered although the total stiffness of the structure may change which in turn may affect the deflections of the structure. In general, in the stiffness matrix of a truss, the diagonal elements are the predominant elements in any row. This is because the member stiffness matrix of a truss has squares of the direction cosines as the diagonal elements which are always positive and the equilibrium equations of a joint which lead to the structure stiffness matrix, adds up all the positive terms for all the members meeting at the joint. These elements appear on the diagonal of the stiffness matrix giving rise to a very strong diagonal. Generally the off diagonal terms are single stiffness elements and are smaller . This clearly shows that for trusses the choice of member areas do not influence the convergence for analysis. The trusses generally have low degress of indeterminacy as the addition of one member only increases the degree of redundancy by one unlike in frames where the addition of a member increases the indeterminacy by three. Since changes in the member forces depend mostly upon the

degrees of indeterminacy : for a truss the member areas do not influence the member forces to any appreciable extent like the frames. Because of this any arbitary areas may be assumed for the members as preliminary areas. The volume of the structure obtained in the first cycle of design and the volume when all members are fully-stressed does not differ much.

Because of the above facts most of the trusses do not need a preliminary design. In the computer programmes developed equal areas for all members are assumed to start with.

If a preliminary design becomes necessary, then a statically determinate truss may be chosen out of the indeterminate structure. Member areas may be determined for the determinate structure and the volume of the truss may be calculated. On the assumption that an indeterminate truss is lighter than the determinate one the above volume or few percent less than the volume may be distributed among all the members of the indeterminate truss according to the reciprocal of member lengths, since the forces carried by a member depends upon the reciprocal of its length.

For the final design the trusses may be classified into three categories.

4.21 Determinate Trusses

The equations of statics are enough for the analysis of these structures. The member forces are independent of the member areas and the preliminary design is the final design. The members may be proportioned to achieve full-stress condition either under one or multiple load conditions.

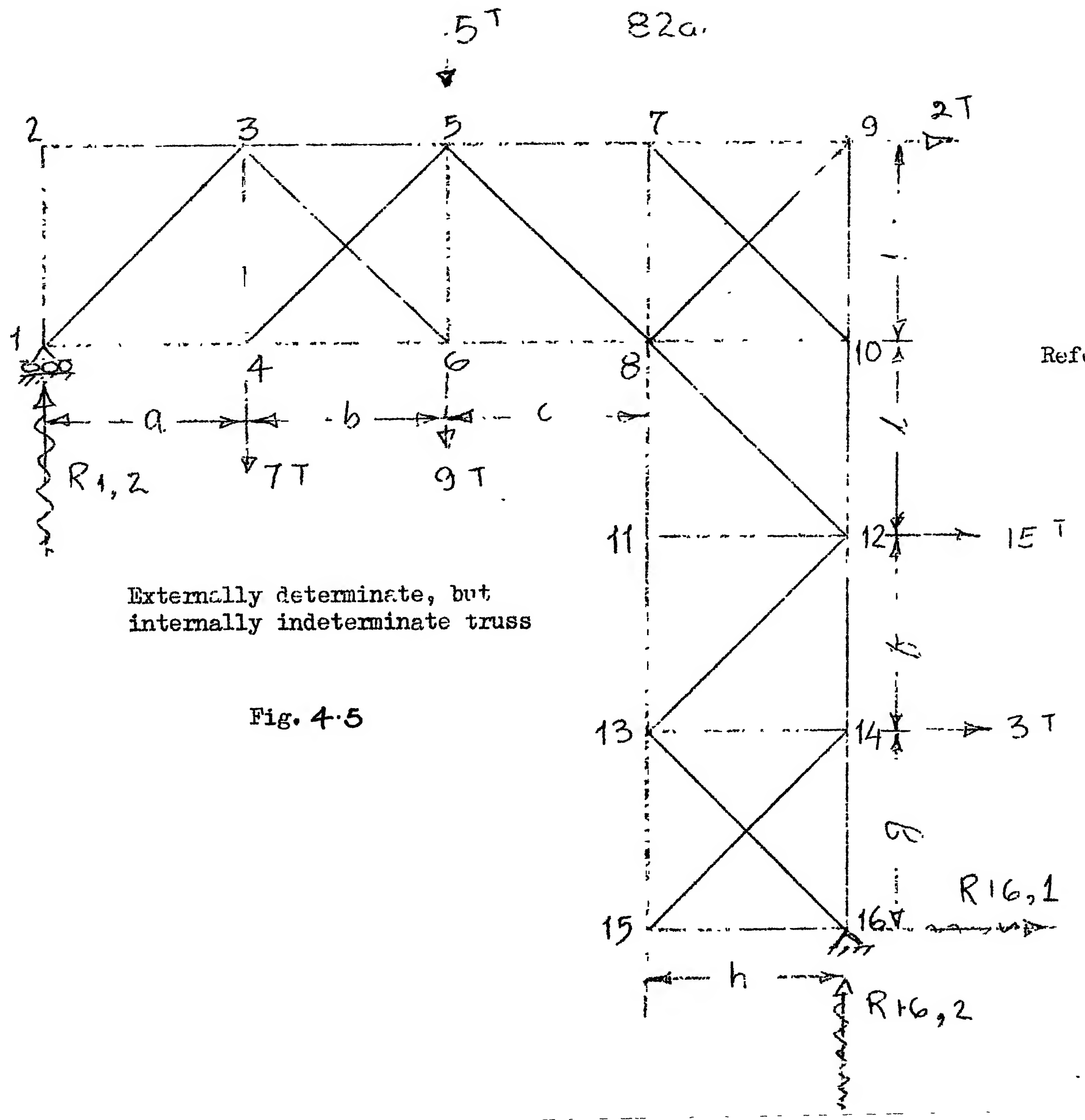
4.22 Internally Indeterminate Trusses

For a structure of this type the external reactions can be determined by the Laws of Statics alone. Since the structure is a determinate one for the external reactions but the internal member forces are indeterminable by statics these type of trusses may be called as internally indeterminate trusses.

For a plane truss three independent equations of statics are available to determine the external reactions and if the reaction components are three then it comes under this catogory. For a space truss these reaction components are six. The above number of reaction components make each truss stable and externally determinate.

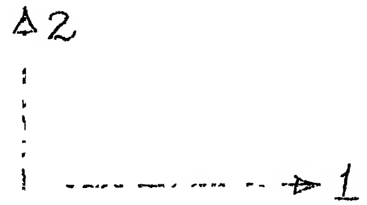
Fig. 3 shows an internally indeterminate truss.

Although it is an indeteminate structure the member forces, for the determinate panels namely panels 1 - 2 - 3 - 4, 5 - 7 - 6 - 8, 8 - 10 - 11 - 12, and 11 - 12 - 13 - 14 can be determined by statics. These forces remain independent of any member areas of the structure.



Externally determinate, but internally indeterminate truss

Fig. 4.5



Reference Coordinate System

For example, taking a section in the panel 5 - 7 - 6 - 8 the member forces can be determined by the method of section as follows:

Taking moment about the joint '8'

$$F_{57} = - R_{12} \times (a + b + c)/d \qquad .. \; 4.33$$

Taking moment about joint '5'

$$F_{68} = R_{12} (a + b)/d \qquad .. \; 4.34$$

Taking moment centre at joint '6'

$$F_{58} = - (R_{12} x(a + b) + F_{57} \times d)/d \qquad .. \; 4.35$$

Similarly forces in the other determinate panels may be evaluated by the equations of statics.

An internally indeterminate truss may be divided into two Complexes.

(a) The Determinate Complex

This includes all the determinate panels. The member forces of this complex only depend upon the external loading and are determinable by statics alone like any determinate structure. The member areas of this complex does not influence the force of any members of the structure. These members may be designed just like statically determinate trusses. The member proportions should be kept constant throughout the design of the structure. This may save lot of human and computer time than designing all the members of the truss for each design cycle.

(b) The Indeterminate Complex

This includes all the indeterminate panels.

To evaluate the forces of members belonging to this complex indeterminate analysis has to be carried out. The forces of members belonging to this complex are influenced by areas of this complex alone. This can be readily arrived at by strain energy approach.

Let $U = U_1 + U_2$ .. 4.36

where U = Total Strain Energy of the whole structure, U_1, U_2, the strain energy of the determinate and indeterminate complexes respectively.

The fundamental equations of Force method are:

$dU/dR_i = r_i$; where R_i is the redundant force and r_i is the displacement along R_i

But $dU/dR_i = dU_2/dR_i$ as $dU_1/dR_i = 0$ as U_2 is ca constant and is independent of R_i as member forces of this complex are statically determinable.

$$dU_1/dR_i = r_i \qquad .. \; 4.37$$

or the strain energy of the indeterminate complex alone determines the redundants.

This complex can separately be dealt and member forces determined. Since the member forces are a function of the member areas of this complex alone at each stage of the design these areas are to be altered till full-stress in all members or minimum volume is achieved.

Many of the bridge trusses contain a few indeterminate panels and come under this category. If the above procedure is adopted designer can complete the design in much

less a time than necessary for the usual procedure in which no demarcation is made about the complexes.

4.23 Externally Indeterminate Trusses

A truss is defined to be externally indeterminate when the external reaction components cannot be determined by the laws of statics alone. If the reaction component exceeds three for a plane truss or six for a space truss then it comes under this category. These reaction components are not only dependent on the external load but also are influenced by the stiffness of all the members of the structure. Since forces on all members depend upon the external reactions member forces are influenced by the stiffness of all the members. The proportioning of all the members has to be carried out at each design cycle.

4.24 Method of Design

In this procedure only indeterminate Complex members in case of an internally indeterminate truss and all members of an externally indeterminate truss are designed. Other members may be proportioned to achieve full-stress just like a determinate truss.

As earlier indicated there are two separate types for the design of indeterminate trusses.

1. Design for Multiple Load Condition
2. Design for Single Load Condition

4.25 Design for Multiple Load Condition

It is earlier pointed out that even for multiple load condition, the compatibility equations may not allow all

the members to be fully stressed. Although this is true, this becomes a rare case. This is because the real multiple load condition (defined in the Chapter - Feasibility of a full-stress design) depends upon the areas of the members of a truss and these areas keep on changing at each design cycle. So even though the area distribution initially may not give a real multiple load condition, the areas can distribute in such a manner that at a subsequent design cycle real multiple load condition may be achieved. Generally except for some "pathological" cases full stress in all members may be achieved for a multiple load condition.

The design for a truss under the above restrictions may be broken up to the following steps.

(1) For an assumed area of the members, the member forces may be evaluated.

(2) For the above analysis proportion the areas of the members to achieve full stress in all the members. In this design, for compression members, compression buckling has to be prevented. The design SUBROUTINE developed for the design of plane frame members may be used if moment and shear forces are equated to zero. The use of lattrice and battened members make it possible to avoid the compression buckling altogether and permissible stresses may be achieved in most of the cases.

(3) For the above areas the analysis may be carried out onee again and member stresses evaluated. The absolute maximum of these stresses is calculated and determine the factor as:

$$\beta = \frac{\text{The absolute maximum stress in any members of the structure}}{\text{The permissible stress}} \quad .. \ 4.38$$

If $\beta > 1$ then some of the members are overstressed and for $\beta < 1$ all member stresses are within the allowable value and it is a feasible design.

(4) For $\beta > 1$ prorate all the areas of the members by the factor β. By this process the analysis remains unchanged since the relative stiffnesses of the members remain the same. This isnow a feasible design.

(5) For $\beta \leq 1$ redesign the members to achieve full stress.

The design is said to be complete when the volume for the feasible designs of steps 4 and 5 has a minimum value or remains constant as the design iteration proceeds.

If the volume for the feasible designs are plotted against the design iteration it is found that either the volume reduces till it remains a constant or it reduces and then increases (see Illustrative problems) and finally remains a constant as iteration proceeds. In the case where the volume

reduction is monotonic the full-stress design is also the minimum weight design otherwise the full stress design is not the minimum weight design.

For the above procedure no mathematical proof is found as yet but the plot of volume vs iteration for some problems gives lesser volume than that of a full-stress design. It should not be thought that full-stress design has approximately the minimum volume as even for a lesser volume than this the relative member areas may be so distributed that the member forces and stresses may be lower than the fully stress values. This may be the real absolute volume of the structure.

4.26 Design for Single Load Condition

In 1900 Celley[46] pointed out that for one load condition generally a determinate structure has the minimum weight. For this weight the deflections are also a minimum. It was also pointed out in the Chapter on the Feasibility of a fully-stressed design that a fully-stressed statically indeterminate structure can not exist for one load condition as it violates the equations of compatibility. So it is not at all desirable to go for an indeterminate truss to support only one set of loads. But if it is necessary to design a statically indeterminate structure may be to minimise the deflections then the same procedure as for multiple load condition may be adopted except that instead of proportioning all the members to attain full-stress limits only M-N members

be designed to achieve full stress,where M is the total number of members and N is the number of indeterminance of the structure. By doing so the equations of compatibility are not violated. Wilber and Norrice[47] have pointed out that for one load condition if a statically indeterminate structure be designed to fully stress all the members at each cycle of design then finally several member areas will reduce to zero to yield a statically determinate structures. This is because compatibility laws are violated. But the recommended design does not give a determinate structure as all the laws of analysis are satisfied.

The choice of (M-N) members out of the M members is is a really difficult problem. This choice may decide the volume of the structure. This (M-N) quantities may be chosen according to the member stresses or member forces. Suppose an indeterminate truss has 15 members and the redundants are five then either choose the 10 highest member forces and proportion these members to achieve full stress or choose 10 highest member stresses and design these members for full-stress. The volume of the structure may be determined for both the cases and the lighter structure may be chosen for economy. This volume will definitely be more than the volume for a determinate structure as essentially we are designing a statically determinate structure out of the total structure and other members of the structure carry little load only due to the laws of compatibility. This structure may have lesser deflections than the determinate one.

In conclusion, it may be pointed out that we are introducing arbitrary means to satisfy the laws of analysis and there is no constraints to minimize the volume at all. So this design may not have the absolute minimum volume;

```
ILLUSTRATIVE PROBLEM NO 1

PROBLEM DESCRIPTION ....PLANE TRUSS ....

  INPUT

 NPROB = 2*

M , N , IN , ND = 4 , 2 , 3 , 2

( JTOT(I),I=1,N )  = 3 , 3
```

I	(JOINT(I,J),J=1,JTOT(I))		
1	2	3	4
2	1	3	4

I	(X(K,I),K=1,ND)	
1	000.0	000.0
2	096.0	000.0
3	096.0	072.0
4	000.0	072.0

LOAD CONDITION	I	(P(I,K),K=1,ND)	
1	1	10.0	15.0
	2	00.0	00.0
2	1	00.0	00.0
	2	20.0	00.0

OUTPUT

MEMBER NUMBER	MEMBER AREA	MEMBER FORCE FOR LOAD CONDITION I	MEMBER STRESS FOR LOAD CONDITION I	MEMBER FORCE FOR LOAD CONDITION II	MEMBER STRESS FOR LOAD CONDITION II	VOLUME
			CYCLE NO 1			
12	0.947	4.706		-7.019		
13	1.357	12.723		8.333		

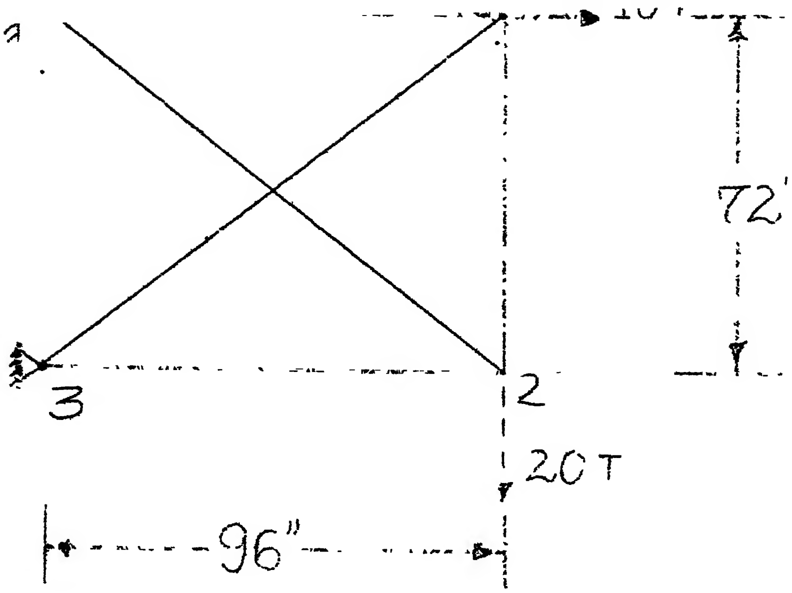
14	0.675	2.366		-7.019		
23	0.974	3.530		9.736		
24	1.623	-5.8833		-16.226		
						567.486

MINIMUM WEIGHT IS ACHIEVED AT THIS CYCLE

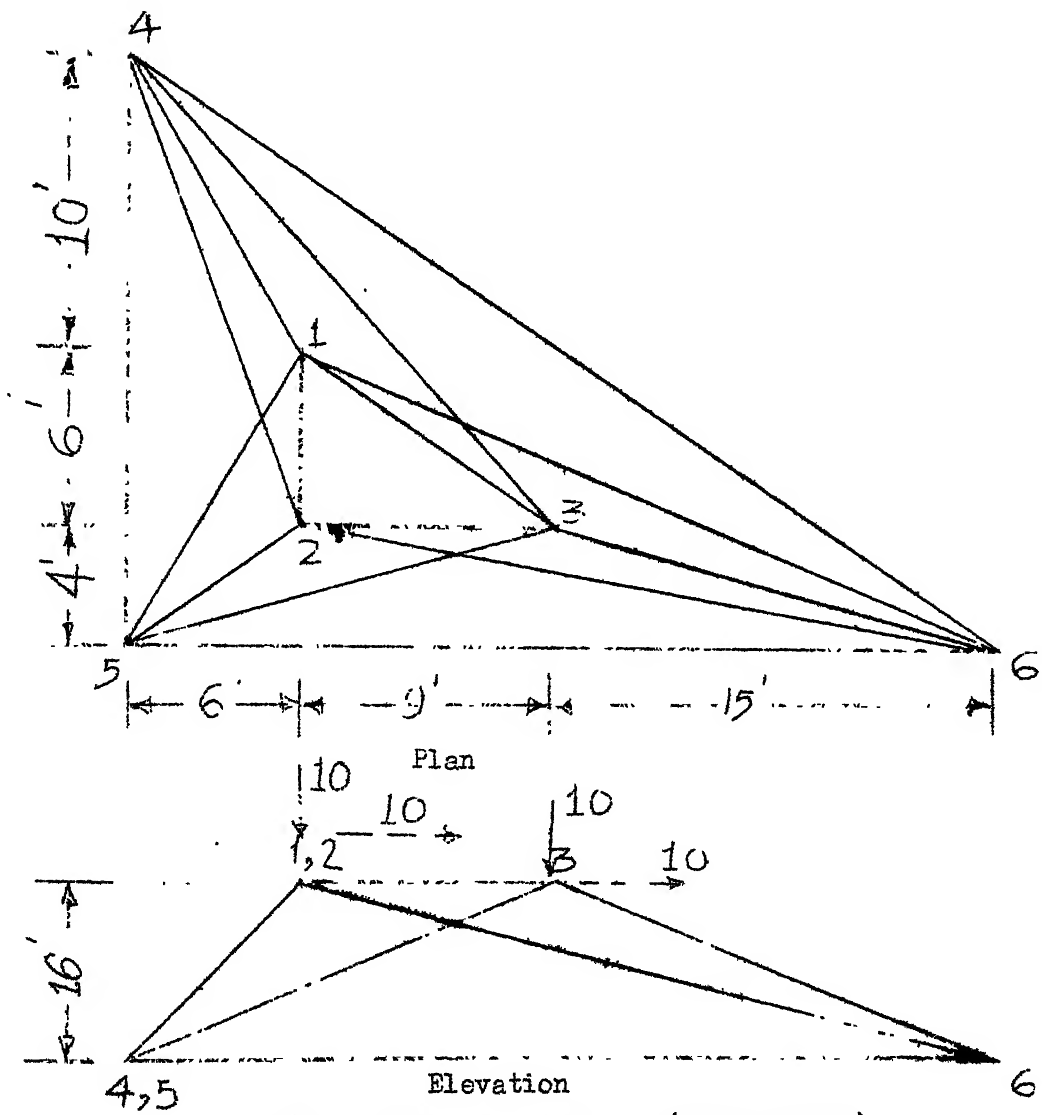
12	0.560	4.068	7.25	-5.111	-9.15	
13	1.368	13.614	10.00	6.201	4.55	
14	0.415	1.832	4.41	-3.720	-8.95	
23	1.117	3.051	2.72	11.167	-10.00	
24	1.861	-5.083	-2.68	-18.611	10.00	
						551.50

FULL STRESS CONDITION IS ACHIEVED AT THIS CYCLE

12	0.418	3.509	8.35	-4.183	-10.00	
13	1.435	14.350	10.00	5.166	3.60	
23	1.186	2.632	2.23	11.860	10.00	
24	1.977	-4.386	-2.20	-19.771	-10.00	
						557.50



ILLUSTRATIVE PROBLEM NO. 1 (PLANE TRUSS)
FIG. 5·1



ILLUST..ATIVE PROBLEM NO. 2 (SPACE TRUSS)
FIG. 5.2

ILLUSTRATIVE PROBLEM NO 2

PROBLEM DESCRIPTION .... SPACE TRUSS NO 1 ....

I N P U T

NPROB = 2

M,N,IN,ND = 6 , 3 , 5 , 3

(JTOT(I),I=1,N) = 5 , 5 , 5 , 5

I	(JOINT(I,J),J=1,JTOT(I))				
1	2	3	4	5	6
2	1	3	4	5	6
3	1	3	4	5	6

I	(X(K,I),K=1,ND)		
1	72.0	120.0	192.0
2	72.0	48.0	192.0
3	180.0	48.0	192.0

4	00.0	240.0	00.0
5	00.0	00.0	00.0
6	360.0	00.0	00.0

LOAD CONDITION	I	(P(I,K),K=1,ND)		
1	1	10.0	00.0	00.0
	2	10.0	00.0	00.0
	3	10.0	00.0	00.0
2	1	00.0	00.0	10.0
	2	00.0	00.0	10.0
	3	00.0	00.0	10.0

MEMBER NUMBER		MEMBER AREA	MEMBER FORCE 6OR LOAD CONDITION I	MEMBER STRESS +009 LOAD CONDITION I	MEMBER FORCE F009 LOAD CONDITION II	MEMBER STRESS FOR LOAD CONDITION II	VOLUMF

MINIMUM VOLUME IS ACHIEVED AT THIS CYCLE

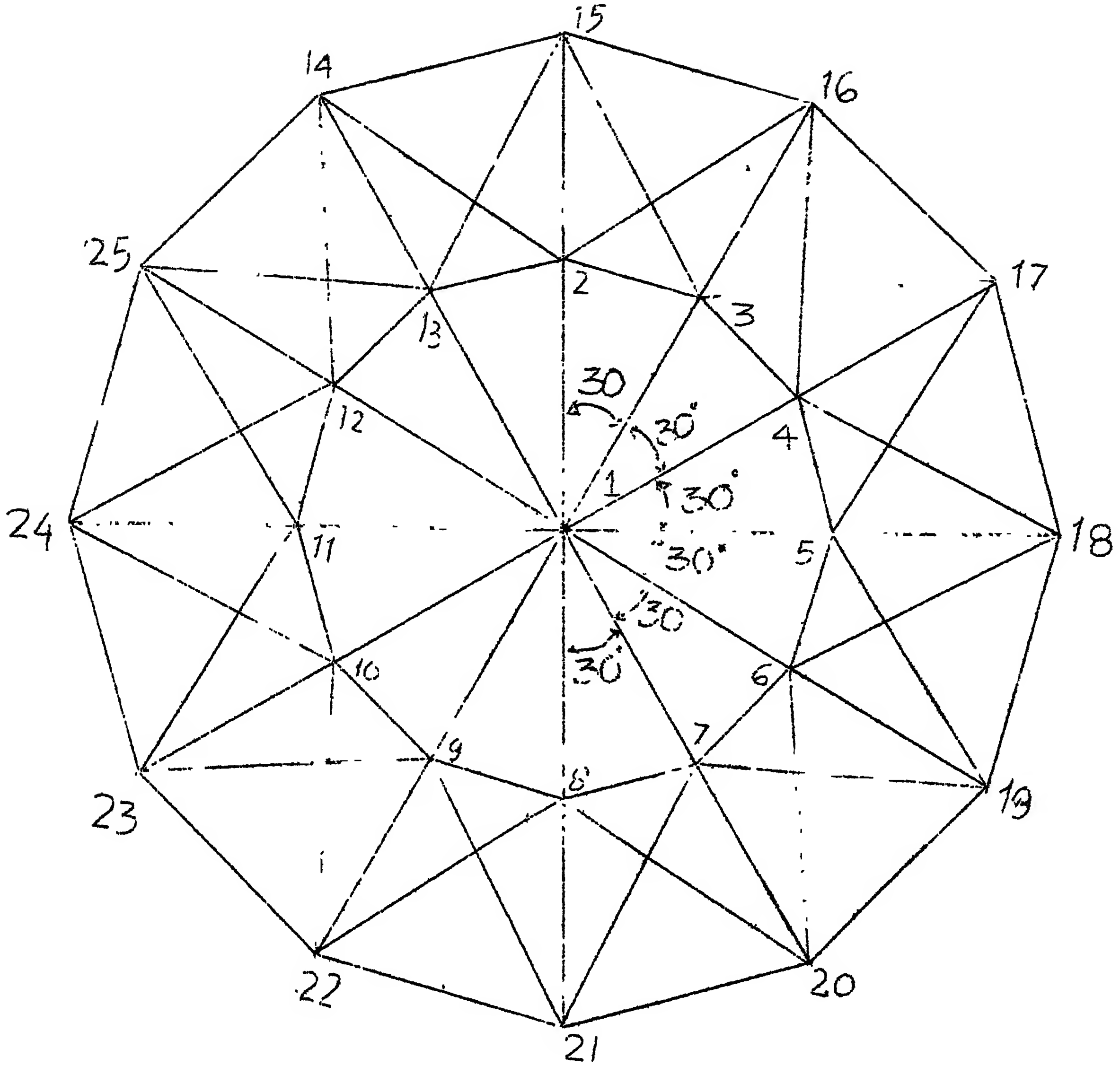
1	2	.50	-.30	-.61	-2.85	-5.72	
1	3	.39	-1.12	-2.91	1.26	3.28	
1	4	.99	-4.98	-5.05	7.44	7.55	
1	5	1.05	-10.46	-9.99	-6.75	-6.45	
1	6	.84	-5.97	-7.15	-1.06	-1.27	
2	3	.29	.16	.53	2.94	9.99	
2	4	.58	-1.41	-2.41	3.33	5.72	
2	5	1.23	-12.12	-9.86	-6.63	-5.39	
2	6	.96	-5.67	-5.93	6.86	7.16	
3	4	1.45	-1.44	-1.00	12.62	8.72	
3	5	1.65	-7.16	-4.35	-13.33	-8.10	
3	6	1.46	-12.51	-8.57	2.96	2.03	2966.41

FULL STRESS IS ACHIEVED AT THIS CYCLE

1	2	.37	3.73	10.01	-.00	-.02
1	3	.71	-7.05	-10.00	.90	1.28
1	4	1.03	3.78	3.81	10.35	10.00
1	5	2.49	-24.90	-10.00	-9.89	-3.97
1	6	.42	-4.20	-9.99	-.76	-1.80
2	3	.56	-1.98	-3.55	5.57	9.99
2	4	0.1	.07	7.0	.10	10.0
2	5	1.71	-17.06	-10.00	-2.60	-1.53
2	6	.43	-3.67	-8.54	4.31	10.01
3	4	1.29	3.30	2.56	12.90	10.00
3	5	1.53	-5.49	-3.59	-15.28	-10.00
3	6	1.84	-18.35	-10.00	4.69	2.56

2999.53

```
ILLUSTRATIVE PROBLEM NO 3

PROBLEM DESCRIPTION .... SPACE TRUSS NO 2 ....

I N P U T

NPROB = 2

M,N,IN,ND=  25 , 13, 5 , 3

  (JTOT(I),I=1,N) = 24 , 9 , 9 ,9 , 9 , 9 , 9 , 9 , 9, 9 , 9 , 9 , 9

  I          (JOINT(I,J),J=1,JTOT(I))

  1       2    3    4    5    6    7    8    9   10   11   12   13
         14   15   16   17   18   19   20   21   22   23   24   25
  2      11   12   13   14   15   16   17   18   19   20
  3       1    2   15   16   17    4    6    9   12
  4       1    3   16   17   18    5    7   10   13
  5       1    4   17   18    1    6    8   11    2
  6       1    5   18   19   20    7    9   12    3
  7       1    6   19   20   21    8   10   13    4
  8       1    7   20   21   22    9   11    2    5
```



Plan

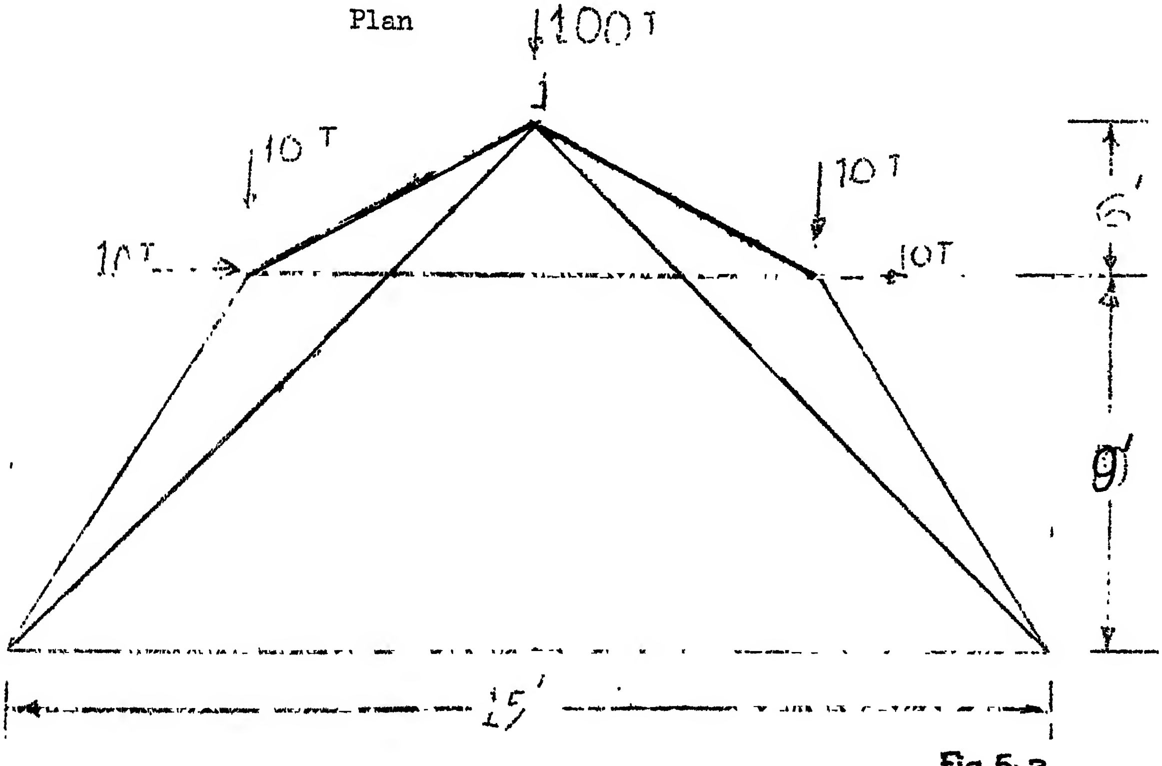


Fig 5·3

ILLUSTRATIVE PROBLEM NO. 3 (SPACE TRUSS)

9	1	8	21	22	23	10	12	3	6
10	1	9	22	23	24	11	13	4	7
11	1	10	23	24	25	12	2	5	8
12	1	11	24	25	14	13	3	6	9
13	1	12	25	14	15	2	4	7	10

I	(X(I,K),K=1,ND)		
1	6.000	10.392	9.000
2	10.392	6.000	9.000
3	12.000	0.000	9.000
4	10.392	-6.000	9.000
5	6.000	-10.392	9.000
6	0.000	-2.000	9.000
7	-6.000	-10.392	9.000
3	-10.392	-6.000	9.000
9	-12.000	-0.000	9.000
10	-10.000	6.000	9.000
11	-6.000	10.39	9.000
12	0.000	12.000	9.000

13	).000	15.000	0.000
14	7.500	12.990	0.000
15	12.990	7.590	0.000
16	15.000	0.000	0.000
17	12.990	7.500	0C000
18	7.500	12.990	0.000
19	0.000	15.000	0.000
20	-7.500	-12.990	0.000
21	-2.990	-7.500	0.000
22	-5.C00	0.000	0.000
23	-12.990	7.500	0.000
24	-7.500	12.990	0.000
25	0.000	0.000	15.000

LOAD CONDITION	I	(P(I,K),K=1,ND)		
1	1	000.000	000.000	100.000
	2	000.000	000.000	10.000
	3	000.000	000.000	10.000
	4	000.000	000.000	10.000

5	000.000	000.000	10.000
6	000.000	000.000	10.000
7	000.000	000.000	10.000
8	000.000	000.000	10.000
9	000.000	000.000	10.000
10	000.000	000.000	10.000
11	000.000	000.000	10.000
12	000.000	000.000	10.000
13	000.000	000.000	10.000
1	000.000	15.000	00.000
2	000.000	15.000	00.000
3	000.000	15.000	00.000
4	000.000	15.000	00.000
5	000.000	15.000	00.000
6	000.000	15.000	00.000
7	000.000	15.000	00.000
8	000.000	15.000	00.000
9	000.000	15.000	00.000

10	000.000	15.000	00.000
11	000.000	15.000	00.000
12	000.000	15.000	00.000
13	000.000	15.000	00.000

MEMLER NUMBEK	MEMBER AREA	MEMBER FORCE FOR LOAD CONDITION I	MEMBER STRESS FOR LOAD CONDITION I	MEMBER FORCE FOR LOAD CONDITION II	MEMBER STRESS FOR LOAD CONDITION II	MEMBER VOLUME
1 2	.42	.32	.77	-4.16	-10.01	
1 3	.59	-5.94	-10.00	-1.44	-2.43	
1 4	.99	-9.85	-9.99	2.25	2.28	
1 5	.57	-5.74	-10.00	2.11	3.67	
1 6	.26	.40	1.56	2.55	10.00	

1	8	.16	.20	1.23	1.63	10.01
1	9	.54	-5.40	-10.00	1.49	2.76
1	10	.87	-8.74	-10.00	-.46	-.53
1	11	.69	-6.86	-10.00	-2.55	-3.72
1	12	.36	-.57	-1.61	-3.56	-10.00
1	13	.41	-.32	-.77	-4.08	-9.99
1	14	1.58	-15.80	-10.00	10.31	6.52
1	15	1.22	-12.21	-10.00	6.87	5.63
1	16	1.09	-10.93	-10.00	3.97	3.63
1	17	1.01	-10.05	-10.00	.20	.20
1	18	.86	-8.62	-10.00	-3.30	-3.83
1	19	.91	-9.13	-10.00	-6.68	-7.32
1	20	1.00	-10.00	-10.01	-8.90	-8.91
1	21	.87	-8.69	-10.00	-6.80	-7.83
1	22	.55	-5.53	-9.99	-2.52	-4.56
1	23	.59	-5.59	-9.48	-.27	-.45
1	24	.70	-6.98	-10.00	2.23	3.20
1	25	1.22	-12.21	-10.00	6.87	5.63
2	3	.62	-.74	-1.20	-6.18	-10.00
2	16	1.33	-5.25	-3.94	-13.33	-10.00

2	15	.03	-.25	-9.84	-.01	-.44
2	14	1.66	-15.66	-9.41	11.25	6.76
2	13	1.22	-3.01	-2.48	-12.17	-10.02
2	11	.39	-2.05	-5.27	-3.89	-9.99
2	8	.22	2.24	10.00	-.34	-1.51
2	5	.38	3.78	10.00	-3.12	-8.24
3	15	.16	-1.55	-10.03	1.05	6.79
3	16	1.36	-13.58	-10.00	10.80	7.95
3	17	1.50	-3.94	-2.64	-14.96	-10.00
3	4	1.27	9.22	7.27	-12.69	-10.00
3	6	.40	4.04	10.00	1.59	3.93
3	9	.00	-.05	-9.40	.01	2.60
3	12	.16	-1.60	-9.82	-1.63	-9.99
4	16	3.19	-16.33	-5.12	31.93	10.00
4	17	.11	-1.14	-10.00	-.16	-1.40
4	18	3.04	-7.64	-2.51	-30.42	-10.00
4	5	.59	5.89	10.00	4.14	7.03
4	7	.62	1.97	3.21	6.15	10.00
4	10	.02	-.19	-10.05	.07	3.63
4	13	.59	1.67	2.86	-5.85	-10.00
5	17	2.51	-1.65	-.66	25.08	10.00
5	18	.22	-2.17	-10.02	-1.50	-6.93

5	19	2.21	-19.29	-8.75	-22.05	-10.00
5	9	1.48	3.81	2.57	14.83	10.00
5	8	.39	-2.84	-7.33	3.37	10.01
5	11	.00	-.02	-9.50	-.00	-1.00
6	18	1.35	-11.53	-10.93	-4.87	10.07
6	20	.90	-9.00	-10.00	-4.01	-4.45
6	7	1.29	-7.39	-5.72	12.91	10.00
6	9	.37	-2.81	-7.68	3.66	10.00
6	12	.27	2.66	10.00	-.26	-.98
7	19	.19	-1.94	-10.02	1.57	8.11
7	20	2.10	-21.00	-10.00	-2.22	-1.06
7	21	.13	-1.29	-9.98	1.11	8.58
7	8	1.37	-7.44	-5.44	13.67	10.00
7	10	.56	1.70	3.05	5.59	10.00
7	13	8.82	8.21	.93	-.62	-.07
8	20	1.01	-10.01	-9.94	-4.33	-4.30
8	21	1.01	-10.70	-10.63	-4.45	-4.42
8	22	1.04	-.63	-.60	10.43	10.00
8	9	1.49	2.48	1.66	14.94	10.00

9	21	2.24	-19.76	-8.84	-22.35	-10.00
9	22	.17	-1.68	-10.00	-1.20	-7.14
9	23	2.46	-1.73	-.70	24.62	10.00
9	10	.36	3.55	10.00	3.45	9.71
9	12	.45	4.48	10.00	-1.93	-4.30
10	22	2.32	-9.06	-3.91	-23.20	-10.00
10	23	.13	-1.27	-10.00	-.01	-.09
10	24	2.29	-14.07	-6.13	22.94	10.00
10	11	.62	6.17	10.00	-4.16	-6.75
10	13	.45	2.24	4.93	-4.54	-10.00
11	23	2.49	3.81	1.53	-24.88	-10.00
11	24	.22	-2.24	-10.00	1.14	5.10
11	25	2.66	-26.58	-10.00	21.65	8.15
11	12	1.42	3.89	2.74	-14.19	-10.00
12	24	.96	1.89	1.96	-9.64	-9.99
12	25	1.09	-10.91	-10.00	3.74	3.43
12	14	1.49	-14.94	-10.00	3.78	2.53
12	13	1.23	-3.88	-3.15	-12.30	-10.00
13	25	.05	-.42	-8.75	-.48	-10.00
13	14	2.68	-26.81	-10.00	1.03	.38
13	15	.05	.42	8.75	-.83	-17.29

ILLUSTRATIVE EXAMPLE NO 7 .... BEAM I ....

PROBLEM DESCRIPTION DESIGN OF A PLANE FRAME ELEMENT

I N P U T

SPAN	=	240.00	INCHES
BENDING MOMENT	=	3084.83	TON-INCH
SREAR FORCE	=	17.42	TONS
AXIAL FORCE	=	8.89	TONS

O U T P U T

DEPTH OF BEAM	=	26.74	INCHES
AREA OF CROSS-SECTION	=	33.62	SQUARE INCHES
MOMENT OF INERIIA OF SECTION	=	4984.31	INCH TO POWER 4
VALUE OF INTER ACTION FORMULA	=	0.96	

ILLUSTRATIVE EXAMPLE NO 8 .... BEAM II

PROBLEM DESCRIPTION DESIGN OF A PLANE FRAME ELEMENT

I N P U T

SPAN	=	300.00	INCHES
BENDING MOMENT	=	1500.00	
SREAR FORCE	=	12.00	TONS

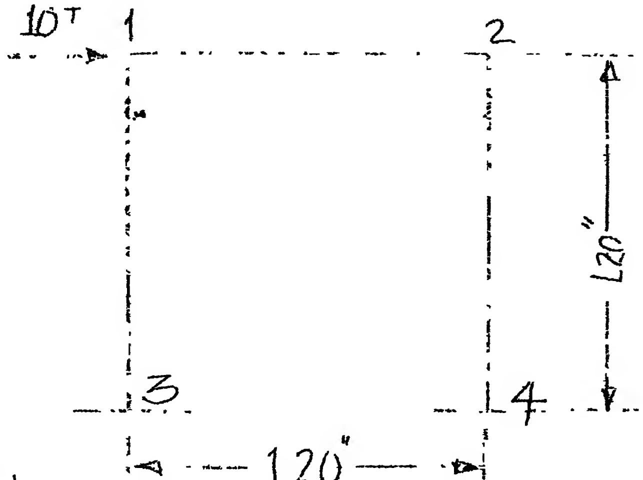
```
AXIAL FORCE                      =    50.00   TONS

O U T P U T

DEPTH OF BEAM                    =    15.40

AREA OF CROSS-SECTION            =   116.40   SQUARE INCHES

MOMENT OF INERTIA OF SECTION     =   707.69   INCH TO POWER 4

VALUE OF INTER ACTION FORMULA    =     1.02
```

```
              ILLUSTRATIVE PROBLEM NO 4

              PROBLEM DESCRIPTION .... PORTAL ....

I N P U T

              M , N , IN , ND  =    4  , 2  , 2  , 2

              ( JTOT(I),I=1,N )=  2 , 2

      ---------- -- -----------------------------------   ---------- --------

              I            ( JOINT(I,J),J=1,JTOT(I))

      ---------- --  ----------------------------------   ---------- --------

              1      2    3

              2      1   4

      -------------  ----------------------------------   ---------- --------

              I      (P(I,J),J=1,3)

      -------------  ----------------------------------   -- ------- --------

              1      00.00      00.00      10.00

              2      00.00      00.00      00.00

      ---------- --  ----------------------------------   ---------- --------

              I      (X(I,K),K=1,ND)

      --------- - --- ---------------------------------   ---------- --------

              1     000.00     000.00
```

2	120.00	120.00
3	000.00	000.00
4	120.00	000.00

O U T P U T

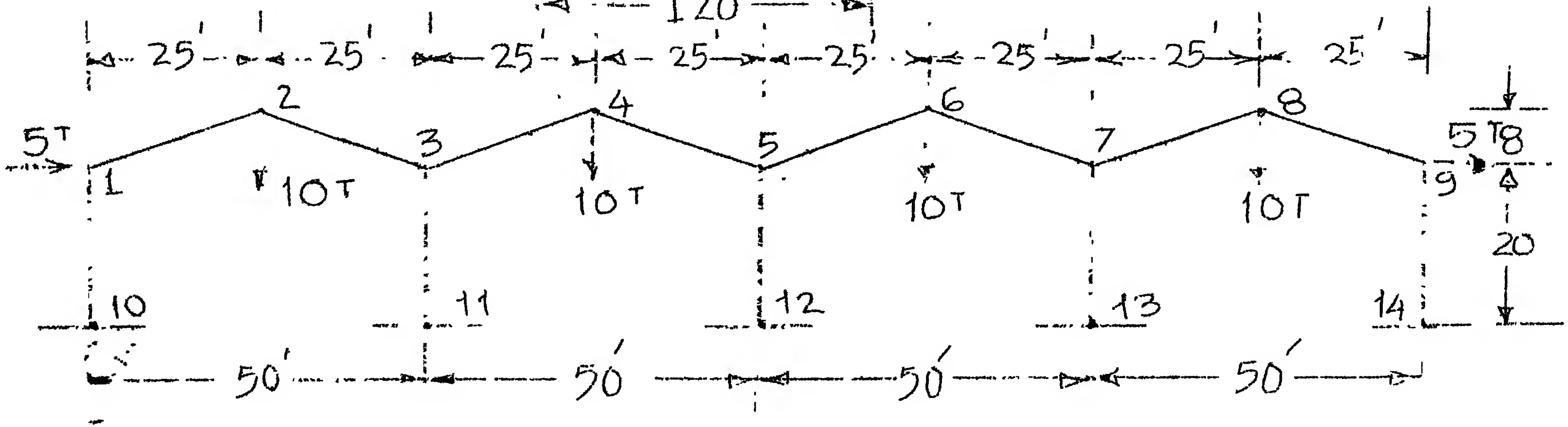
MEMBER NUMBER		MEMBER DEPTH	MEMBER AREA	MEMBER MOMENT OF INERTIA	INTERACTION RELATION
			CYCLE NO 4		
1	2	5.55	5.76	32.76	.96
1	3	5.55	7.91	47.00	3.00
2	4	5.55	8.07	47.70	.95
			CYCLE NO 5		
1	2	5.55	5.89	33.70	.95
1	3	5.80	7.90	50.10	.96
2	4	5.83	7.90	50.30	.95



ILLUSTRATIVE PROBLEM NO. 4

PORTAL FRAME

Fig. 5·4



ILLUSTRATIVE PROBLEM NO. 5

GABLE BENT

Fig. 5·5

```
            ILLUSTRATIVE PROBLEM NO 5

            PROBLEM DESCRIPTION .... GABLE FRAME ....

   I N P U T

       M , N, IN , ND  =  14    9    3    2

       ( JTOT(I),I=1,N )  = 2    2    3    2    3    2    3    2    2
---------------------------------------------------------------------

       I    (JOINT(I,J),J=1,JTOT(I))
---------------------------------------------------------------------

       1    10    2
       2     1    3
       3     2   11    4
       4     3    5
       5     4   12    6
       6     5    7
       7     6   13    8
       8     7    9
       9     8   14
```

I	(X(I,K),K=1,ND)	
1	000.0	240.0
2	300.0	336.0
3	600.0	240.0
4	900.0	336.0
5	1200.0	240.0
6	1400.0	336.0
7	1800.0	240.0
8	2100.0	336.0
9	2400.0	240.0
10	000.0	000.0
11	600.0	000.0
12	1200.0	000.0
13	1800.0	000.0
14	2400.0	000.0

I	(P(I,J),J=1,3)		
1	00.0	00.0	5.0
2	00.0	15.0	00.0
3	00.0	00.0	00.0
4	00.0	15.0	00.0
5	00.0	00.0	00.0
6	00.0	15.0	00.0
7	00.0	00.0	00.0
8	00.0	10.00	00.0
9	00.0	00.0	5.0

MEMBER NUMBER	MEMBER DEPTH	MEMBER A-EA	MEMBER MOMENT OF INERTIA	INTERACTION -+LATION

MEMBER PROPORTION AT 7TH CYCLE OF DESIGN

1	10	26.40	34.16	5072.53	.95
1	2	16.08	20.50	1083.81	.99
2	3	13.22	13.84	481.30	.96
3	11	10.08	8.70	166.17	1.00
3	4	13.22	15.08	530.48	1.00
4	5	13.22	14.93	524.79	1.03
5	12	9.60	3.00	39.92	.57
5	6	13.22	15.45	544.85	.98
6	7	13.22	15.42	543.95	.98
7	8	13.22	11.95	404.99	.99
7	13	10.58	10.16	220.14	1.04
8	9	13.22	13.54	469.23	1.01
9	14	12.86	12.71	414.12	1.02

MEMBER PROPORTION AT 6TH CYCLE OF DESIGN

1	10	26.74	34.05	5055.18	.96
1	2	16.08	19.60	1032.83	1.00
2	3	13.23	13.94	485.23	.99
3	11	10.08	8.60	164.38	.96
3	4	13.22	15.17	533.95	.96
4	5	13.89	14.91	574.28	1.05
5	12	9.60	3.00	39.92	.57
5	6	13.22	15.05	529.28	1.02
6	7	13.22	15.06	529.58	1.02
7	13	10.08	10.96	219.90	.95
7	8	13.22	11.89	402.71	.96
8	9	13.22	14.10	4.91	8.00
9	14	13.50	13.70	494.93	.96

ILLUSTRATIVE PROBLEM NO.7

PROBLEM DESCRIPTION.... PLANE ARCH....

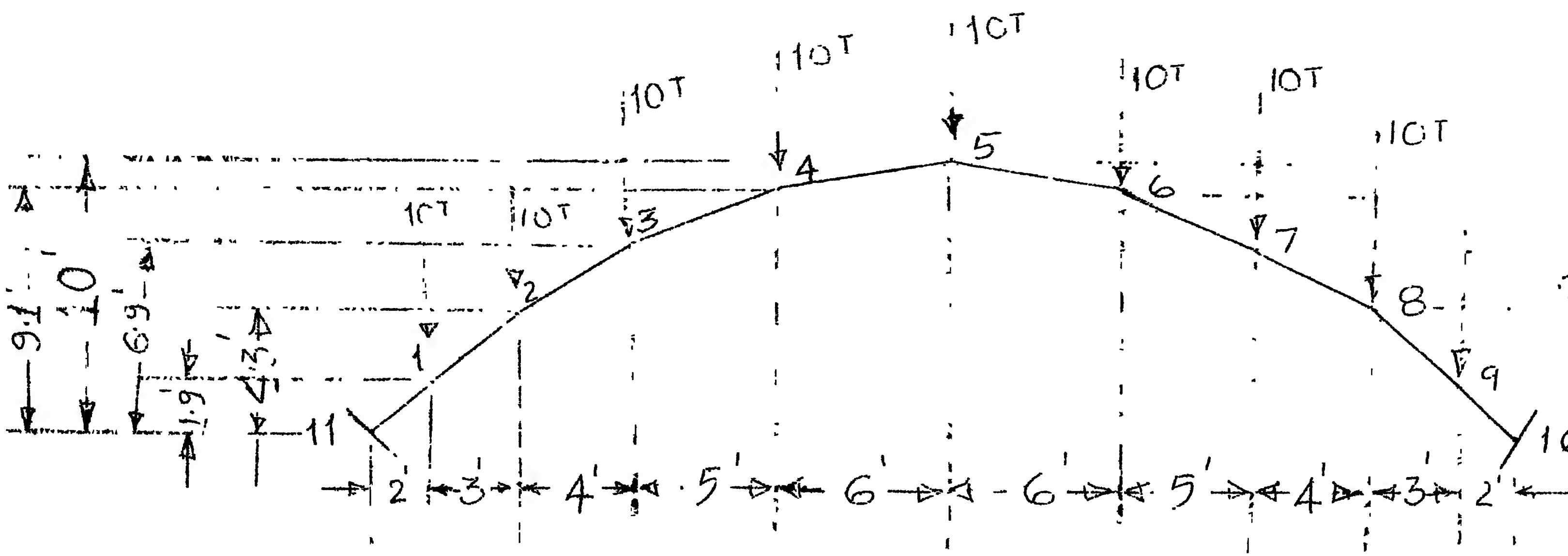
I N P U T

M,N,IN,ND = 11, 9, 2, 2

(JTOT(I), I=1, N) = 2, 2, 2, 2, 2, 2, 2, 2, 2

I	(JOINT(I,J), J=1, JTOT(I))	
1	11	2
2	1	3
3	2	4
4	3	5
5	4	6
6	5	7
7	6	8
8	7	9
9	8	10

RIGID JOINTED PLANE ARCH



Illustrative Problem No. 7

Fig.

I	(P(I,J),J=1,3)			(X(I,J), J=1,ND)	
1	00.0	10.0	00.0	2.000	1.900
2	00.0	10.0	00.0	5.000	4.370
3	00.0	10.0	00.0	9.000	6.960
4	00.0	10.0	00.0	14.000	9.100
5	00.0	10.0	00.0	20.000	10.000
6	00.0	10.0	00.0	26.000	9.100
7	00.0	10.0	00.0	31.000	6.960
8	00.0	10.0	00.0	35.000	4.370
9	00.0	10.0	00.0	38.000	1.900
10				40.000	0.000
11				00.000	0.000

O U T P U T

MEMBER NUMBER	MEMBER DEPTH	MEMBER AREA	MEMBER MOMENT OF INERTIA	INTERACTION RELATION
		MEMBER PROPORTION AT 5TH CYCLE OF DESIGN		
11 - 1	30.01	50.80	10787.52	1.048
1 - 2	28.30	44.20	9832.10	0.952
2 - 3	24.69	41.30	7615.21	0.952
3 - 4	22.40	40.06	4279.00	0.952
4 - 5	22.40	40.22	4297.12	0.951
5 - 6	22.40	40.22	4298.01	0.953
6 - 7	22.40	40.30	4299.12	0.952
7 - 8	24.63	41.30	7616.21	0.952
8 - 9	28.10	44.50	9831.10	0.951
9 - 10	30.20	50.90	10788.10	0.981

At this cycle all member properties have conversed so next cycle value are not reproduced.

CHAPTER VI

SUMMARY OF RESULTS, CONCLUSIONS AND DISCUSSIONS

Since the process of resisting external loads for rigid jointed and pin-jointed structures are different, these structures are separately treated to highlighten the important features involved in their design.

6.1 Pin-jointed structures

Reference : Chapter V : Illustrative Problem No. 1

This is a plane truss with one degree of indeterminacy and is subjected to two load conditions. It took six cycles to achieve the stationary values of the joint displacements by the point iteration technique with an initial assumed zero displacements for all joints. All the member areas were assumed to be equal at the outset. The first feasible solution was obtained by prorating all the areas of the members in accordance with the maximum member stress. This volume was 567.48 cubic inches. The members were then proportioned to achieve full stress for all members in either of the load conditions. For these member areas and joint displacements, only three cycles of iterations were required for the point iteration procedure to converge. The member stresses determined and member areas prorated to achieve another feasible solution. Graph 5.1 shows the volume for feasible structures versus the cycles of iteration. The minimum volume was obtained at the sixth iteration. For this volume only

CHAPTER VI

SUMMARY OF RESULTS, CONCLUSIONS AND DISCUSSIONS

Since the process of resisting external loads for rigid jointed and pin-jointed structures are different, these structures are separately treated to highlighten the important features involved in their design.

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three of the five members were fully-stressed under either of the load conditions. The full-stress condition was obtained at the 15th cycle with a volume of 557.5 cubic inches. The difference between the first cycle volume and the minimum volume was found to be 3% of the economic volume.

Reference: Chapter V : Illustrative Problem No. 2.

This is a space truss with three degrees of indeterminancy and is designed for two load conditions. The first feasible volume of the structure was found to be 3680 cubic inches. Graph 5.2 shows the volume for feasible structures against cycles of design. For this structure also the full stress volume was obtained as 2999.53 cubic inches whereas the minimum volume was obtained at the second iteration as 2966.41 cubic inches. At this minimum weight only one of the members is fully-stressed and all others are stressed lower than the permissible stress. The difference between 1st cycle volume and the economic volume is 24% of the minimum volume.

Reference: Chapter V : Illustrative Problem No. 3.

This is a space truss and represents a hemi spherical dome in geometry. Since it is a ring structure the structure stiffness matrix does not have a band. It has a total of 91 members and the degree of statical indeterminacy is 47. For this structure the feasible areas decreased from a value of 21000 to 12760 cubic inches

representing a volume change of 65%. For this structure at minimum weight all the members are fully stressed at either of the load condition.

From the above design examples, the following may be concluded.

(1) At minimum weight all the members of the structure need not be fully-stressed. A feasible solution is achieved by iterating and prorating the areas. L.S. Schmit (2) arrived at the same conclusion by solving the problems using mathematical programming.

(2) Since the stiffness matrix of a truss have always strong diagonal elements the rate of convergence to achieve the correct solutions is quite fast. For subsequent cycles of design where the structure stiffness matrix does not change much it requires only two to three cycles for most of the cases for a stationary value of joint displacements. This advantage cannot be taken of in the direct procedure where the stiffness-matrix is inverted to get the joint displacements. So although the point iteration may take a little longer time to achieve the joint displacements for the first cycle, for subsequent cycles, it may be faster than direct inversion. For example, for the space truss of figure 5.3 the total structure stiffness matrix will have a dimension of 39 x 39. For joint displacement this 39 x 39 matrix is to be inverted. By point iteration only 13; 3 x 3 matrices are inverted. If for subsequent cycle

an average of two iterations are necessary the total number of 3 x 3 matrices inverted are 26 where as in the direct inversion they are two 39 x 39 matrices.

(3) By this procedure only storage required for the stiffness matrix of the structure are two 3 x 3 or 2 x 2 joint stiffness matrix for space and plane frame. This saves computer memory to a great extent and even a small computer like IBM 1620 can handle structures with hundreds of degrees of freedom. This is especially true for problems like Illustrative problem 3 of Chapter 5 where there is no band formation in the stiffness matrix.

(4) It is seen that the percentage difference between the initial design volume and final design volume differs according to the degrees of indeterminacy. This is expected as member areas are influenced by redundants.

(5) Proration of areas gives several feasible solutions and generally takes less number of cycles to achieve the minimum weight conditions than necessary to achieve a full-stress design. For problem number 1, number of design cycles were only 6 and for problem 2 it is only 2 to achieve the minimum weight where as the full stress design takes fifteen cycles for problem no. 1 and over 30 cycles for problem 2. But for structures where fully-stress design is also the minimum weight design, such conclusions cannot be applied.

(6) For externally determinate structures by going through the design of members of the indeterminate complexes computer time may be saved.

Although the illustrative examples are all indeterminate structures the computer programme developed can also handle determinate structures.

The general computer programme developed solves both space and plane trusses, the second as a special case of the first. The dimension of the structure is represented by ND which takes the value 2 for a plane truss and 3 for a space truss. The programme is in terms of ND and either a plane or a space truss is analysed according to if ND is read as 2 or 3. The number of statements of the programme for joint displacements are only 39 plus ten or five according to whether it is a space or a plane truss.

6.2 Rigid jointed Plane Frames

Reference : Chapter V : Illustrative Problem No. 4

This is a square portal, subjected to a horizontal load of 10 tons at the floor level. Without the introduction of initial joint deformation and for arbitary member proportions the convergence for sway displacement was extremely slow although the rotations and axial deformations converged within first four cycles of iteration. The sway deformations of the joints are achieved through the axial deformation of the beams. So with an arbitary area of the beam the process of convergence became extremely slow. For example the final sway deformation , was built up from zero by $P_i L/AE$ the axial deformation of the beam where P_i is the unbalanced sway force at the ith cycle of iteration and these are related by the relation

$$\triangle = P_i L/AE \text{ or } (L/AE) \sum_{i=1}^{k} P_i \qquad .. \ 6.2$$

where k is the total cycles needed for stationary values of the joint deformation.

It is well-known that the area of the beams do not influence the final sway deformations of joints to any appreciable order of magnitude. The sway deformation primarily depends upon the moment of inertia of column group of a storey. So this is a very peculiar process as the

convergence of sway displacements but not the final sway displacement depend upon the area ofthe beams. So with an arbitary area of the beams the process of convergence of sway deformations may greatly be influenced.

The initial sway deformations and member sizes are calculated by approximate methods. When these initial guess values for sway deformation and member proportions are assumed then it took very few cycles for the convergence. For example the approximate sway deformations was 0.60 inches where as the actual sway deformation was 0.64 inches. The final deformation was found on the higher side because the beams are assumd relatively rigid than the columns in the cantilever analysis.

The design area of members are determined at their economic depths and for a particular set of loads and a span there are several areas but the minimum one was achieved by iteration on depth. For columns this iteration converged within four to five cycles where as for beam it takes 5 to 15 cycles to converge depending primarily upon the bending moments moments.

For this case the column proportions converged with in the second design cycle within an accuracy of 5% whereas the beams took two more cycles to converge.

The sway deformation at both the ends of the beam (namely at joint 1 and 2) remain same except for a

small percentage (5%) due to the axial deformation of the beams.

Reference : Chapter V : Illustrative Problem No. 5

This is a continuous gable with four bays and is subjected to both wind loads and gravity loads.

The joint deformations for this problem converged faster than the portal. This is because it has several inclined members and the off diagonal term $6EI/L^2$ in the shear equation which caused poor convergence was distributed throughout the shear equation which improved the convergence of joint displacements.

Like the previous example to achieve the economic depths the beams take more number of cycles than the columns.

In the design cycle all but the first and last column size converged within three cycles whereas the beams took 5 cycles.to converge. This is because both these columns carry high moments as the horizontal loads are concentrated at these member ends. Both fifth and sixth cycle member sizes are given in the illustrative chapter 5.

Reference : Chapter V : Illustrative Problem No. 6

This is a five bay and five storey building frame. The process of achieving the final design was completely automated. Only the number of bays, number of

storeys and the span of each bay, height of each storey, the joint loads and joint coordinates are the input data. Rest of the calculations are done within the computer.

For this problem the sway deformation introduced by approximate method was helpful but the final and initial values different by 100% for the top floor as assumed sway for top floor was 0.0015 whereas actual sway was 0.0029 inches. For bottom floor the respective values are 0.00039 and 0.00099. These are small values and are due to sway and axial deformation of beams. Hence although the results for this problem does not look encouraging, since these are small values and their contribution to the force equilibrium equations are not significant, the convergence was faster. For example all the deformations converged within the first 20 cycles.

In the design cycle the column convergence was found to be faster than the convergence of beams. The initial design and final design agree for columns within 40% whereas beam sizes change from 50 to 100% between preliminary design and first cycle design.

From the above design examples the following may be concluded.

(1) As already noted for trusses the storage is only two 3 x 3 stiffness matrices for any type of plane frames. By this method even using a small computers like IBM 1620,

structures with hundreds of degrees of freedom, can be solved.

(2) Like trusses structures without a proper band formation can be solved in this method whereas the solution of such types are highly involved by regular procedure.

(3) The operations and storage in a computer can still be economized if sway for a storey is assumed constant for all the joints of the storey and axial deformation of columns are neglected. Columns carry predominantly the axial load and if the axial stresses are same in magnitude and sense for all the columns of a storey the strain and also the axial deflections of all the columns of the storey with uniform column heights will be same. This means the beams will sink uniformly creating no beam forces due to axial deformation. But in case of wind loads where columns are in tension such conclusions cannot be drawn and axial deformations are to be considered. In the design, the elements have the absolute minimum weight under the loads. This saves the weight to a high degree. The depth of an element solely depends upon the forces it has to carry. The depth of 1/10 of the span generally assumed, although may be correct for quite a few cases but may be uneconomical for several other cases. For example for beam 1 (Fig.) for a depth of 1/10 the span the area is 35 square inches whereas the minimum area is 33.62 square inches at a depth of 26.74 inches. For this case the design office assumption

is correct. But for beam 2 the minimum area is 16.4 sq. inches at a depth of 15.4". If a depth of 1/10th the span is assumed, the area is as high as 19.58 sq. inches. So in this case the increase in area is nearly 26%. So if the areas are calculated at the economic depths 20% of the tot frame weight may be saved.

Once the section areas and moment of inertia are thus obtained the sections can either be fabricated or be selected from handbooks. Economic factors like fabrica tion should govern the selection.

This way of design of a structures has two distinct advantages.

(1) It saves material as the elements are designed for minimum area.

(2) The structure designed is safe and at no place will be overstressed. In the usual design where a structure is analysed and the members are proportioned for the analysis the final design stresses may not be same as the assumed stresses and hence in some cases overstress of the member may result.

6.3 Future Work on this Field

The structures designed are elastic. But as load increases some of the mebers come to the plastic zone. So instead of going on an elastic path, the stress-strain curve may be assumed non-linear and at the member load

proper elastic modulus 'E' may be taken according to the actual case. This will change only the SUBROUTINE STIFIJ and STIFJI which calculate the element stiffness matrix in the local coordinate system.

Once the member sizes are determined, these members may either be fabricated or for the areas and moment of inertia standard sizes may be taken from proper handbooks. The section which gives maximum economy may be the governing criterion. For example if the fabrication cost is higher these standard sizes should be used otherwise they may be fabricated.

CHAPTER VII

EXPLANATION OF COMPUTER PROGRAMMES AND FLOW CHARTS

MAIN PROGRAMME

INPUT

(i) Geometry of the Frame

The total number of joints of the frame; the number free joints; Number of Bays and Floors; Height of each Storey and Span of each Bay; Coordinates for all joints.

(ii) External Loads

Forces acting at each joint,load and type of load along the span of each member.

EXECUTION

CALL APPROX.*

Analyse the structure by stiffness method. For each joint the joint stiffness matrix is generated. The simultaneous equations are solved by point iteration procedure and joint displacements are determined. The above procedure is repeated for all joints. Structure equilibrium is checked at each cycle and iteration is said to be stationary when the structure equilibrium is satisfied. Member end forces are calculated and design forces determined by CALLING FUNCTION COUPLE, SFORCE and AXIAL. DESIGN SUBROUTINE is CALLED and all members are designed. The structure is reanalysed for these member sizes. This iteration is repeated until the member sizes remain constants at two consecutive cycles.

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* APROX is a SUBROUTINE, the function of it is summarized under the SUBROUTINE Catalogue.

OUTPUT

Depth, Areas, Moment of inertias for each member and the value of interaction formula.

CATALOGUE OF SUBROUTINES

SUB 1: SUBROUTINE: A P R O X

INPUT

The number of Storeys and Storey heights.

Number of Bays and Span of each Bay.

Number of joints and coordinate for each joint.

Sway forces at each storey level.

Load and Load type on the Span of each Beam and Column.

EXECUTION

Calculates the number of members meeting at a joint and the far end joint numbers for each joint.

Calculates the Bending Moment, Shear Force, Axial Force for each member due to wind forces assuming the building as a Cantilever and the same due to gravity loads assuming the beams to be pinned to the Columns at both the ends for Column loads and completely fixed at both the ends for beam loads. Adds up both the effects and determines the member

end moments, shear forces and axial forces. The design forces for each member is obtained by using Bending Moment (FUNCTION COUPLE), Shear Force (FUNCTION SFORCE) and Axial Force (FUNCTION AXIAL) diagrams.

Each member is proportioned for minimum area by CALLING the DESIGN SUBROUTINE.

For the above member forces and sizes, the sway deformations are calculated (FUNCTION DELTA) at each floor level assuming floors to be rigid and the appropriate joints are assigned these values.

OUTPUT

The number of members meeting at a joint and the far end joint numbers for each joint; the areas, moment of inertia for each member and the sway deformations for each joint.

SUB 2: SUBROUTINE: D E S I G N

INPUT

Design Moment, Shear and Axial Force; Span; and permissible Stresses in bending, axial and shear.

EXECUTION

Assumes a depth 1/20 of the Span to satisfy serviceability. For this depth designs an I-

criteria. Changes the depth by 5% and repeats the above cycle until the economic depth is reached, i.e. the depth at which the areas starts increasing.

OUTPUT

Member area, Moment of inertia, Depth and the Interaction Value for the section.

SUB 3: SUBROUTINE: M A T M P

INPUT

Matrix A and Matrix B

EXECUTION

Multiplies A and B and stores in matrix C

OUTPUT

Matrix C

SUB 4: SUBROUTINE: M A R T I N

INPUT

DENOM, a, 3 x 3,Matrix

EXECUTION

Inverts DENOM and Stores it in DINV Matrix

OUTPUT

Matrix DINV

SUB 5: SUBROUTINE: T R A N S

INPUT

The Direction Cosine of any member.

EXECUTION

Generates the TRANSFORMATION Matrix ST for the member.

OUTPUT

Transformation Matrix ST.

SUB 6: SUBROUTINE: X L O A D

INPUT

Joint Loads, Span Load, Joint Coordinates, Total number of members meeting at any joint and the far end joint numbers for each joint.

EXECUTION

Determines the equivalent joint loads due to inspan load and then adds this to the loads directly applied to the joints to give the Joint Load Vector.

OUTPUT

Equivalent JOINT LOAD VECTOR

SUB 7: SUBROUTINE: S T I F I J

INPUT

Member Areas, Moment of inertias, Span and Young's Modulus of the member material, near and and far end joint numbers of the member.

EXECUTION

Generates SK(K_{ij}) matrix in the local coordinate.

OUTPUT

SK Matrix

SUB 8: SUBROUTINE: S T I F J I

INPUT

Same as SUBROUTINE STIFIJ

EXECUTION

Generates SSK(K_{ji}) matrix in the local coordinates

OUTPUT

SSK Matrix

SUB 9: FUNCTION: D E L T A

INPUT

Bending Moment, Span and moment of inertia of members, generally the Columns and Young's modulus of the material.

EXECUTION

Determines the individual SWAY Deformation as $6\ EI/L^2$

OUTPUT

Sway Deformation

SUB 10: FUNCTION: C O U P L E

INPUT

Member End Moments, Shear Forces, and Axial Forces. Type of Load on Span, and distance, x, to a section from the near end A for which all forces are known.

EXECUTION

Determines the bending moment at any section within the Span.

OUTPUT

Bending moment at any section along the span.

SUB 11: FUNCTION: A X I A L

INPUT

Same as FUNCTION COUPLE

EXECUTION

Determines the axial force at any section within the span.

OUTPUT

Axial Force at any Section.

SUB 12: FUNCTION: S F O R C E

INPUT

Same as FUNCTION COUPLE

EXECUTION

Determines the Shear Force at any section along the span.

OUTPUT

Shear Force at any section.

COMPUTER PROGRAMME FOR TRUSSES

INPUT

Total number of joints of the frame. Dimension of the frame; Number of free joints; Number of Boundary joints and degrees of freedom at each Boundary joint; Joint Coordinates; Joint Loads; Far end joint number for each joint.

EXECUTION

For all members meeting at a joint generates the span, the direction cosine and transformation matrix 'T'and the product (T^T T). Member stiffness is determined by multiplying the T^T T matrix by AE/L.

The process is repeated and joint equilibrium equations are generated using the above member stiffness matrix according to the relation 2.11 and joint displacements calculated according to equation 2.12.

The process is repeated for all joints several times till thejoint displacements have a stationary value.

Member end forces determined according to equation 2.14. The above procedure is repeated for all load conditions.

For multiple load condition member areas are prorated according to the maximum member stress if this is more than the permissible value otherwise members are proportioned to achieve full stress.

For single load condition repeat the above procedure accept that only (M-N) member are fully stressed.

OUTPUT

Member area, and member stresses, volume of structure for each design cycle.

FLOW CHART FOR THE DESIGN OF A RIGID JOINTED PLANE FRAME

START

M, N, IN, ND, IFLOOR, IBAY, FORCE, P, WT, X,

COLUMH, DL, ASPAN, SPREAD

INITIALIZATION

U = 0

UO = 0

GENERATE JTOT AND JOINT MATRICES

FOR LATERAL LOAD ANALYSE THE STRUCTURE BY CANTILEVER METHOD AND FOR GRAVITY LOAD ANALYSE IT ASSUMING BEAMS SIMPLY SUPPORTED FOR COLUMN REACTION AND RIGIDLY FIXED FOR BEAM MOMENTS; DETERMINE DESIGN LOAD

CALL DESIGN SUBROUTINE

FOR THE MEMBER SIZES; DETERMINE
APPROXIMATE SWAY DEFORMATION

ANALYSE THE STRUCTURE BY POINT
ITERATION OF THE STIFFNESS
EQUATIONS

1

I = 1, N

INITIALIZATION
INITIALIZATION

DENOM = 0

UPPER = 0

2

J = 1, JJ

2

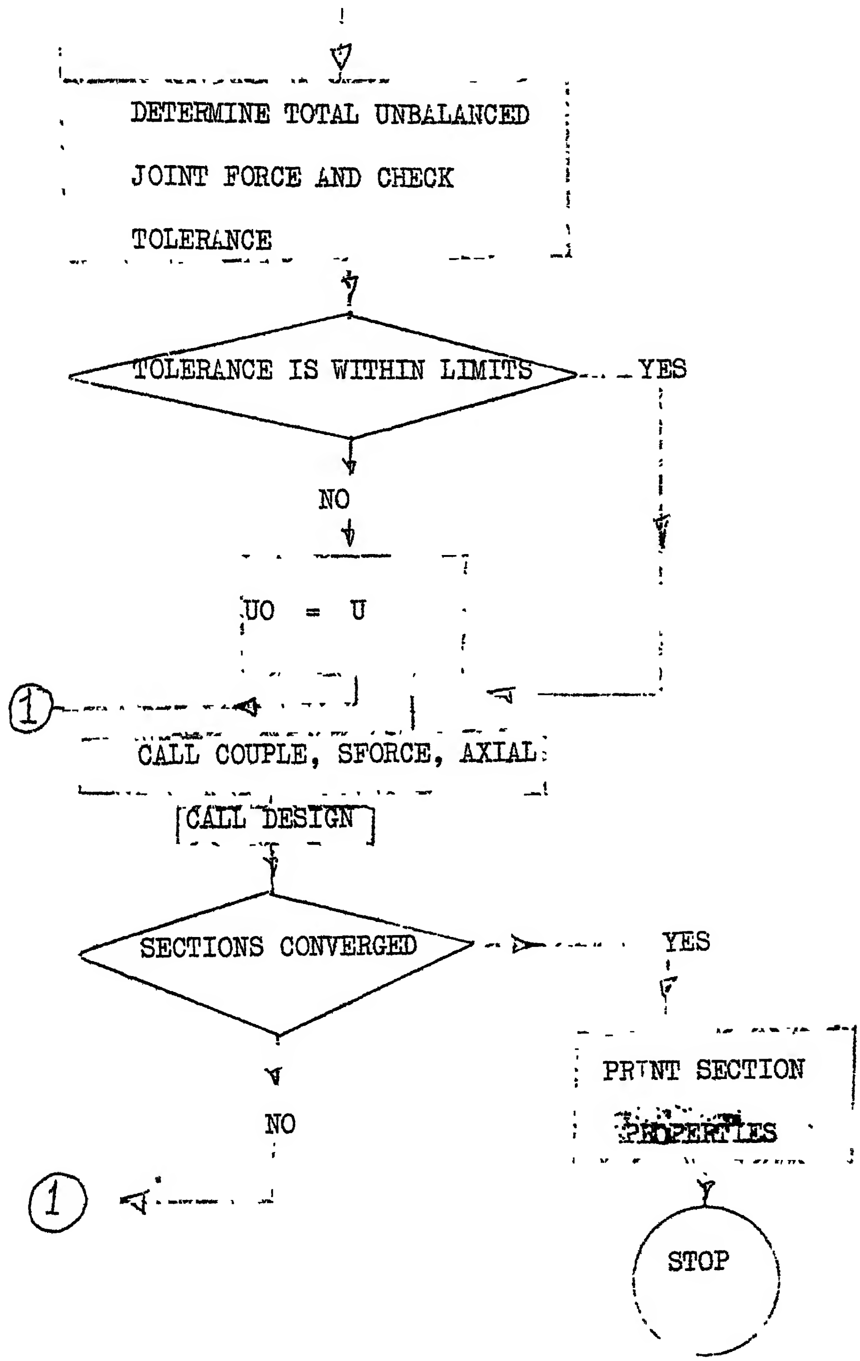
GENERATE SPAN, TT, K_{ij}; K_{ji}, ST, SST Matrices

DENOM $= \sum$ SST K_{ij} ST; UPPER $= \sum$ SST K_{ji} ST D(J)

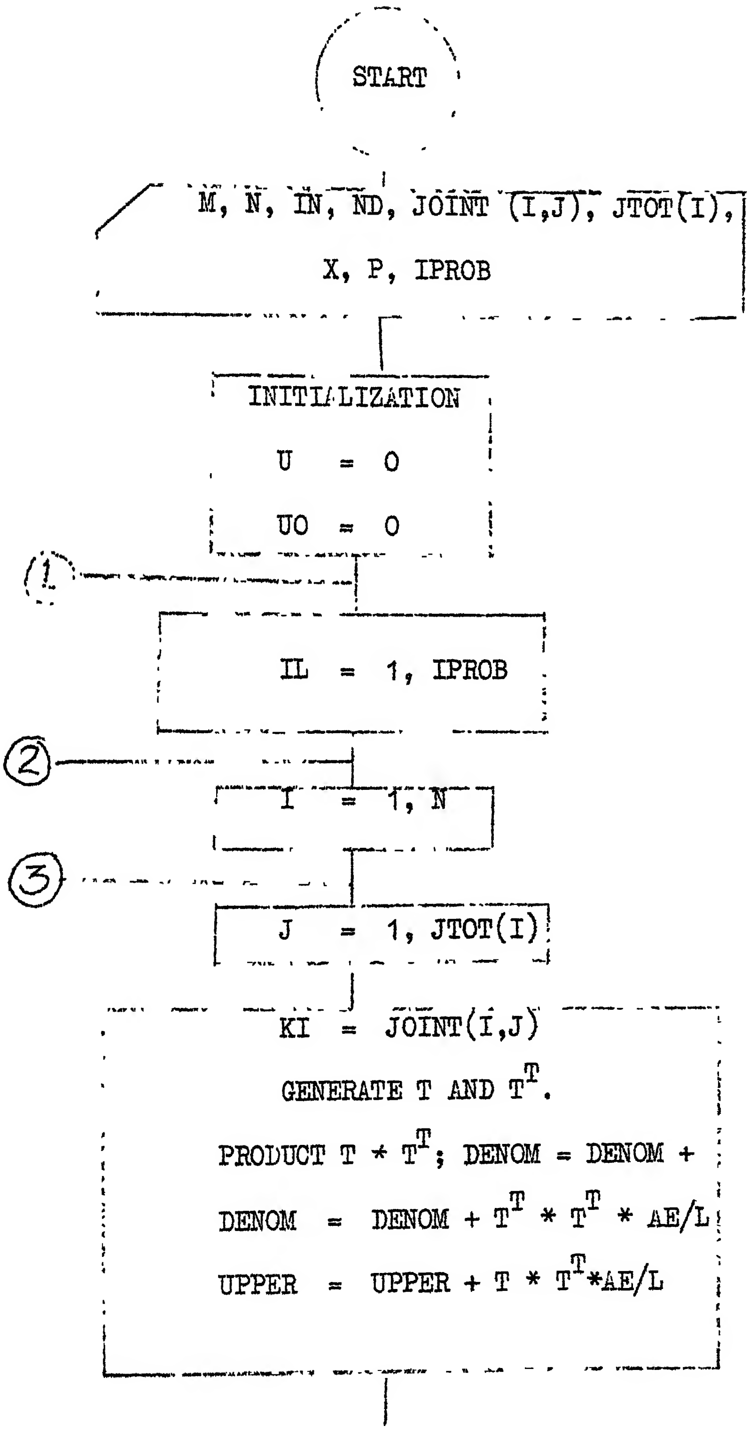
INVERT DENOM AND STORE IN DINV

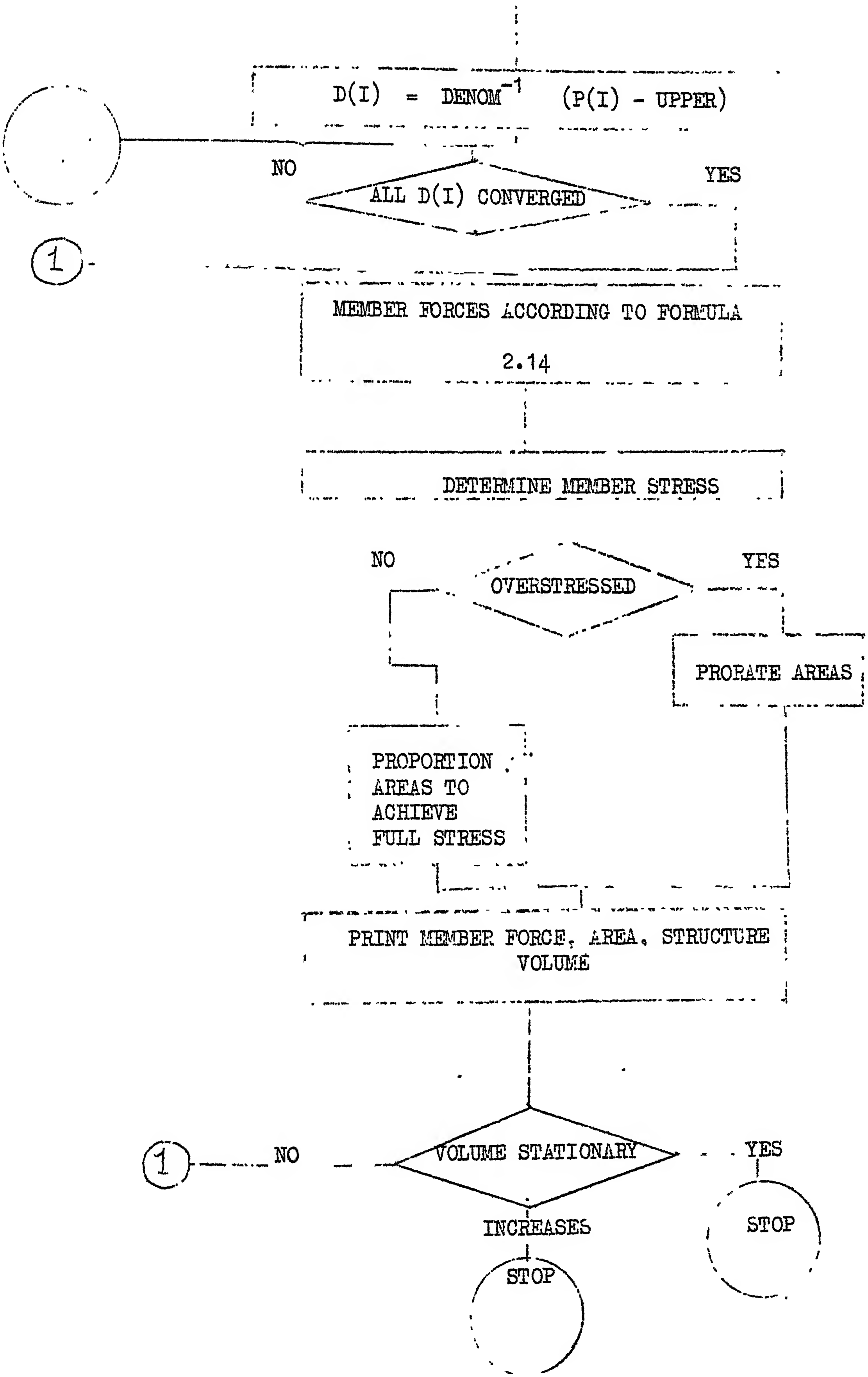
D(I) = DINV * (P(I)-UPPER)

1,2


DETERMINE TOTAL UNBALANCED JOINT FORCE AND CHECK TOLERANCE
TOLERANCE IS WITHIN LIMITS
YES
NO
UO = U
1
CALL COUPLE, SFORCE, AXIAL
CALL DESIGN
SECTIONS CONVERGED
YES
NO
1
PRINT SECTION PROPERTIES
STOP

COMPUTER PROGRAMME FOR PIN CONNECTED FRAMES




D(I) = DENOM^{-1} (P(I) - UPPER)
NO
ALL D(I) CONVERGED
YES
1
MEMBER FORCES ACCORDING TO FORMULA
2.14
DETERMINE MEMBER STRESS
NO
OVERSTRESSED
YES
PRORATE AREAS
PROPORTION
AREAS TO
ACHIEVE
FULL STRESS
PRINT MEMBER FORCE, AREA, STRUCTURE
VOLUME
1
NO
VOLUME STATIONARY
YES
STOP
INCREASES
STOP

NOTATIONS USED IN COMPUTER PROGRAMMES

A, AAREA, AREA	-	Area of the members.
AIXX	-	Moment of inertia of members.
ALFA	-	Tolerance matrix for load
ASPAN	-	Span of the Bays
AX	-	Axial Force
AXMAS	-	Design value of axial force
BBB	-	Intermediate matrix used to convert local in span load to global joint load
BEAMM	-	Moment of beam due to wind load.
BEAMS	-	Shear force of Beam due to wind load
BEND	-	Fraction of bending stress used in the interaction formula
BETA	-	The interaction ratio.
BFL	-	Ld/bt
BMA	-	Bending moment at end A
BMB	-	Bending moment at end B
BRAT	-	Ratio (Ld/bt)/600
CAIXX	-	Column moment of inertia.
CAREA	-	Column area
CG	-	Centre of gravity of the column group of a storey.
CHS	-	Column unit shear force
CO	-	Bending moment at any section of a member.

COLD	-	Column axial force due to gravity load.
COLF	-	Column axial force due to gravity load.
COLM	-	Column moment.
COLS	-	Column shear force.
COLUMH	-	Height of columns.
COLUN	-	Column normal force.
COMAX	-	Design moment for any member.
COUPLE	-	Function which draws bending moment diagram.
DELTA	-	Sway displacement.
DEN		Intermediate matrices in the generation
DENO		of total denominator
DENOM		
DEPTH	-	Depth of section.
DIF, DIFFE	-	Difference of joint displacement between two consecutive cycles.
DINV	-	Inverse of joint stiffness matrix K_{ij}
DL	-	Dead load on span.
DP	-	Maximum depth to prevent bending buckling of the flanges of a beam.
E, EA	-	Youngs modulus of the material.
ERA, ERB	-	Axial force at end A and B of a member.
F_1, F2	-	Intermediate matrices for evaluating member end forces.

FA2	-	Flange area for both flexure and axial force.
FAL	-	The axial stress to prevent compression buckling.
FEA, FEB	-	Fixed end moments for end A and B.
FEM	-	Fixed end moment.
FLANG	-	Design area of flange.
FMAX	-	Design thickness of flange.
FORCE	-	Sway force at storey height.
FWIDTH	-	Design width of flange.
IBAY	-	Number of Bays.
IBL	-	Beam number at left hand side.
IBR	-	Beam number at right hand side.
IBEAM	-	Total number of beams.
ICOLUM	-	Total number of columns.
ICYCLE	-	Iteration cycle for design.
IDOWNL	-	Left hand side joint number for storey below.
IDOWNR	-	Right hand side joint number for storey below.
IFIELD	-	Total number of columns at any storey.
IFLOOR	-	Total number of floors.
IN	-	Maximum number of members meeting at any joint.
ISTRYL	-	Left hand side number of the joint of a storey.

ISTRYR - Right hand side number of the joint of a stress.

ITERA - Iteration cycle for analysis.

IUPL - Left hand side joint number for storey above.

IUPR - Right hand side joint number for storey below.

JOINT - Far end joints for any joint.

JTOT - Total number of members meeting at any joint.

KI - One far end joint.

M - Total number of members including boundary joints.

ML - Top number of a column.

MM - Bottom number of a column.

N - Total free joints of the structure.

ND - Dimension of structure.

P - Loads at joints.

PP, PPR - Fixed edge forces in the member in system coordinate.

RMIN - Minimum radius of gyration.

RXX - Radius of gyration along X axis.

RYY - Radius of gyration along Y-axis.

S - Difference of joint coordinates of a member.

SALOW	-	Allowable shear stress in the web of a beam.
SF	-	Unit shear force.
SFORCE	-	Function which gives the shear force diagram.
SH	-	Shear force at any point of a nmember.
SHEAR	-	Shear force at any storey level.
SHMAX	-	Design shear force.
SIGMA	-	Permissible stress in bending or axial compression.
SK	-	K_{ij}-matrix in local system.
SPAN	-	Span of member.
SPREAD	-	Span of beams.
SSK	-	K_{ij} matrix in local coordinate system.
ST	-	System transfer matrix.
STT	-	Transpose of transfer matrix.
SWAY	-	Sway displacements.
T	-	Direction cosines of a member.
TF	-	Thickness of flange.
TFMAX	-	Design thickness of flange.
TOLRNC	-	Allowed tolerance in the equilibrium equations.
TOTLOD	-	Total load acting in the structure.
TT	-	Direction cosine of any member.
TW	-	Thickness of web.

TWMAX - Design thickness of web.

U - Joint displacements.

UK, UKK - Matrix used to generate the numerator of the joint stiffness matrix.

UNBL - Total unbalanced load at any cycle.

UO - Initial displacements.

UPP
UPPE
UPPER - Same as UK, UKK
UPPT
UU

VALOW - Allowable shear force in the web of a beam.

WW - Span load.

X - Joint coordinates.

ZI, ZXX - Moment of inertia of a member.

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